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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M.Tech (Integrated)

SEM: III - CARRY OVER THEORY EXAMINATION - APRIL 2023

Subject: Discrete Structures

Time: 3 Hours

Max. Marks: 100

General Instructions:

IMP: Verify that you have received the question paper with the correct course, code, branch etc.

1. This Question paper comprises of **three Sections -A, B, & C.** It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.

2. Maximum marks for each question are indicated on right -hand side of each question.

3. Illustrate your answers with neat sketches wherever necessary.

4. Assume suitable data if necessary.

5. Preferably, write the answers in sequential order.

6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A

20

1. Attempt all parts:-

1-a. The binary relation $U = \Phi$ (empty set) on a set $A = \{11, 23, 35\}$ is ____ (CO 1) 1

(a) Neither reflexive nor symmetric

(b) Symmetric and reflexive

(c) Transitive and reflexive

(d) Transitive and symmetric

1-b. For two sets $C = \{3,4,5,6\}$ and $D = \{1,2,3,7\}$ the set $(C - D) \cap D$ will set. (CO1) 1

(a) $\{ \}$

(b) $\{2,3,4,5\}$

(c) $\{2,4,5,6\}$

(d) $\{1,3,4,7\}$

1-c. Let '*' be a binary operation on N defined by $a*b = a-b+ab^2$, then find $4*5$. (CO 2) 1

(a) 9

(b) 88

(c) 98

- (d) 99
- 1-d. Let $(A, \otimes) = (\{1, 2, 3, 4, 5, 6\}, \otimes)$ is a group. It has two sub groups X and Y. $X = \{1, 3, 6\}$, $Y = \{2, 3, 5\}$. What is the order of union of subgroups? (CO2) 1
- (a) 65
- (b) 5
- (c) 32
- (d) 18
- 1-e. Boolean algebra can be used _____ (CO3) 1
- (a) For designing of the digital computers
- (b) In building logic symbols
- (c) Circuit theory
- (d) Building algebraic functions
- 1-f.indicates an upper bound of A that precedes all other upper bounds of A. (CO3) 1
- (a) Sup
- (b) Inf
- (c) Sub
- (d) Super
- 1-g. Which of the following statement is a proposition? (CO 4) 1
- (a) Get me a glass of milkshake
- (b) God bless you!
- (c) What is the time now?
- (d) The only odd prime number is 2
- 1-h. Name of $\vee$ is? (CO4) 1
- (a) Disjunction
- (b) Conditional
- (c) Biconditional
- (d) Implication
- 1-i. What is true about star tree? (CO5) 1
- (a) A tree having n vertices arranged in a line
- (b) . A tree which contains n vertices and n-1 cycles
- (c) A tree having a single internal vertex and n-1 leaves
- (d) A tree which has 0 or more connected subtrees

- 1-j. Which of the following tree data structures is not a balanced binary tree. (CO5) 1
- (a) AVL Tree
 - (b) Red Black Tree
 - (c) B Tree
 - (d) Splay Tree

2. Attempt all parts:-

- 2.a. What is domain and range of a relation? (CO1) 2
- 2.b. In a group $(G, *)$, Prove that the inverse of any element is unique. (CO2) 2
- 2.c. Draw Hasse diagram for $(\{3,4,12,24,48,72\}, /)$ (CO3) 2
- 2.d. Prove that if x is rational and $x \neq 0$, then $1/x$ is rational. (CO4) 2
- 2.e. What is homeomorphism in a graph? (CO5) 2

SECTION B

30

3. Answer any five of the following:-

- 3-a. Explain (i) Operations on Set with suitable example. (ii) Let $A = \{3,4,5\}$ and $B = \{6,8,9,10,12\}$. Let R be a relation from A to B . Find R . (CO1) 6
- 3-b. Show that the function $f(x) = x$ from the set of real numbers to the set of nonnegative real numbers is not invertible, but if the domain is restricted to the set of nonnegative real numbers, the resulting function is invertible. (CO1) 6
- 3-c. Let H be a subgroup of a group G . Suppose that for each element $x \in G$, we have $x^2 \in H$. Then prove that H is a normal subgroup of G . (CO2) 6
- 3-d. Let G, G' be groups. Let $\phi: G \rightarrow G'$ be a group homomorphism. Then prove that for any element $g \in G$, we have $\phi(g^{-1}) = \phi(g)^{-1}$. (CO2) 6
- 3.e. Prove that a non empty finite partial ordered set has: (1). At most one greatest element (2). At most one least element. (CO3) 6
- 3.f. Use De Morgan's laws to find the negation of each of the following statements. (CO4) 6
- a) Jan is rich and happy.
 - b) Carlos will bicycle or run tomorrow.
- 3.g. Define binary Search tree? Explain with example. (CO5) 6

SECTION C

50

4. Answer any one of the following:-

- 4-a. Prove the Identity law and Idempotent law of algebraic structure for Union and also write differences between them. (CO1) 10
- 4-b. Explain following proof techniques: (i) Direct proof (ii) Indirect proof (iii) Proof 10

by induction. (CO1)

5. Answer any one of the following:-

- 5-a. (i) Explain Ring and Field with suitable example. (ii) State and Proof Lagrange's Theorem. (iii) Show that $(\mathbb{N}, +)$ is Monoid. (CO2) 10
- 5-b. Let group $G = (\mathbb{Z}, +)$ and $H = 2\mathbb{Z}$, find the distinct right cosets of H in G . (CO2) 10

6. Answer any one of the following:-

- 6-a. Prove that Product of two lattice is also a lattice with proper justification . (CO3) 10
- 6-b. Prove that a non empty finite partially order set has at most one greatest element and at most one least element. (CO3) 10

7. Answer any one of the following:-

- 7-a. Let $N(x)$ be the statement "x has visited North Dakota," where the domain consists of the students in your school. Express each of these quantifications in English. (CO4) 10
- a) $\exists x N(x)$,
b) $\forall x N(x)$,
c) $\neg \exists x N(x)$,
d) $\exists x \neg N(x)$,
e) $\neg \forall x N(x)$.
- 7-b. Let $Q(x)$ be the statement " $x + 1 > 2x$." If the domain consists of all integers, what are these truth values? (CO4) 10
- a) $Q(0)$,
b) $Q(-1)$,
c) $Q(1)$,
d) $\exists x Q(x)$,
e) $\forall x Q(x)$,
f) $\exists x \neg Q(x)$,
g) $\forall x \neg Q(x)$.

8. Answer any one of the following:-

- 8-a. Explain Prim's method to find the minimum spanning tree of a graph. Illustrate it using an example. (CO5) 10
- 8-b. Define the chromatic number of a graph. What is four color conjecture? Discuss with example. (CO5) 10