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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M.Tech (Integrated)

SEM: II - CARRY OVER THEORY EXAMINATION - MAY 2023

Subject: Engineering Mathematics-II

Time: 3 Hours

Max. Marks: 100

General Instructions:**IMP:** Verify that you have received the question paper with the correct course, code, branch etc.1. This Question paper comprises of **three Sections -A, B, & C.** It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.

2. Maximum marks for each question are indicated on right -hand side of each question.

3. Illustrate your answers with neat sketches wherever necessary.

4. Assume suitable data if necessary.

5. Preferably, write the answers in sequential order.

6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A**20****1. Attempt all parts:-**1-a. The P.I. of the differential equation $(D^2 + 1)y = \sin x$ (CO 1)

1

(a) $(-x/2) \cos x$

(b) $(x/2) \cos x$

(c) $(x/2) \sin x$

(d) $(x/4) \sin x$

1-b. On putting $x = e^z$, the transformed differential equation $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = x$ is (CO 1)

1

(a) $\frac{d^2y}{dz^2} - y = e^z$

(b) $\frac{d^2y}{dz^2} + y = e^z$

(c) $\frac{d^2y}{dz^2} - 2y = e^z$

(d) $\frac{dy}{dz} - 2y = e^z$

1-c.

For the series $\sum_{n=1}^{\infty} u_n$ of positive terms the Raabe's test if $\lim_{n \rightarrow \infty} n \left(\frac{u_n}{u_{n+1}} - 1 \right)$ is (CO2) 1

- (a) < 1
- (b) > 1
- (c) $= 1$
- (d) none of these

1-d. The Fourier coefficient b_1 in a series of sines of multiples of x which will represent x in the interval $(0, 2)$ is (CO2) 1

- (a) $\frac{4}{\pi}$
- (b) $\frac{-4}{\pi}$
- (c) 0
- (d) none of these

1-e. Laplace transform of the function $F(t) = \sinh 3t$ is (CO3) 1

- (a) $\frac{3}{s^2 - 9}$
- (b) $\frac{3}{s^2 + 9}$
- (c) $\frac{1}{s^2 + 9}$
- (d) $\frac{s}{s^2 + 9}$

1-f. Laplace transform of $t^3 e^{-3t}$ is (CO 3) 1

- (a) $\frac{6}{(s+3)^4}$
- (b) $\frac{6}{(s-3)^4}$
- (c) $\frac{3}{(s-3)^4}$
- (d) None of these

1-g. Find the directional derivative of $f(x, y, z) = xy + yz + zx$ in the direction of vector $\hat{i} + 2\hat{j} + 2\hat{k}$ at the point $(1, 2, 0)$ is (CO4) 1

- (a) 3
- (b) 0
- (c) $\frac{10}{3}$

- (d) None of these
- 1-h. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $\text{Curl}\vec{r}$ equal to (CO4) 1
- (a) $\vec{0}$
- (b) 2
- (c) 1
- (d) None of these
- 1-i. Share ₹ 4200 among Jay, Sanjay and Vijay in the ration 2: 4: 6. Find the amount received by Sanjay? (CO5) 1
- (a) ₹1200
- (b) ₹1300
- (c) ₹1400
- (d) ₹1500
- 1-j. A sum of money at simple interest amount to Rs 1045 in 5 years and to Rs 1111 in 6 years. The sum is (CO5) 1
- (a) Rs 945
- (b) Rs 715
- (c) Rs 845
- (d) Rs 775

2. Attempt all parts:-

- 2.a. Find the particular integral of differential equation $(D^2 - 6D + 9)y = \exp(3x)$. (CO 1) 2
- 2.b. Find Fourier coefficients a_n for $f(x) = e^{-x}$, $-1 < x < 1$ (CO2) 2
- 2.c. Find Laplace transform of the function $F(t) = t^2 \sin at$. (CO 3) 2
- 2.d. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, and $r = |\vec{r}|$ show that $\text{div}(\frac{\vec{r}}{r}) = \frac{2}{r}$. (CO4) 2
- 2.e. Divide ₹ 600 among A, B and C so that ₹ 40 more than $\frac{2}{5}$ th of A's share, ₹ 20 more than $\frac{2}{7}$ of B's share and ₹ 10 more than $\frac{9}{17}$ th of C's share are all equal. What is A's share? (CO5) 2

SECTION B

30

3. Answer any five of the following:-

- 3-a. Solve the differential equation $(D^4 - 1)y = \cos x \cosh x$. (CO 1) 6
- 3-b. Find the solution of the differential equation $y'' + y = 0$ satisfying the conditions $y(0) = 1, y(\pi/2) = 2$. (CO 1) 6
- 3-c. Test the Series 6

$$1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots \quad (\text{CO2})$$

3-d. Obtain the Fourier series for $f(x) = \left(\frac{\pi - x}{2}\right), 0 < x < 2$. (CO2) 6

3.e. Find the inverse Laplace Transform of the function $f(s) = \frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$. (CO 3) 6

3.f. The acceleration of a particle at time t is given by $\vec{a} = 18 \cos 3t \hat{i} - 8 \sin 2t \hat{j} + 6t \hat{k}$. If the velocity \vec{v} be zero at t = 0 then find \vec{v} at any point. (CO4) 6

3.g. (i) A boy is now 'a' years old and his father is '5a' years old. How old will the father be when the boy is '3a' years old and how old was the father when the boy was born? (ii) Find the fourth proportional to 5,8,15? (CO5) 6

SECTION C

50

4. Answer any one of the following:-

4-a. Solve the following differential equation by changing the independent variable $\frac{d^2y}{dx^2} - \frac{1}{x} \frac{dy}{dx} + 4x^2y = x^4$. (CO 1) 10

4-b. Solve $(D^2 - DD' - 2D'^2)z = (y-1)e^x$. (CO 1) 10

5. Answer any one of the following:-

5-a. Obtain the Fourier Series to represent the function $f(x) = |x|$ in the interval $-\pi \leq x \leq \pi$. Hence Show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$. (CO2) 10

5-b. Obtain the Fourier series for the function $f(x) = \begin{cases} 0, & -\pi \leq x \leq 0 \\ \sin x, & 0 \leq x \leq \pi \end{cases}$. (CO2) 10
Hence show that $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots = \frac{1}{2}$

6. Answer any one of the following:-

6-a. Find Laplace transform of the function $f(t) = \begin{cases} 1 & 0 \leq t < 2 \\ -1 & 2 \leq t < 4 \end{cases}$ Where $f(t+4) = f(t)$. (CO 3) 10

6-b. Solve the following simultaneous differential equation by using Laplace transform $\frac{dx}{dt} + y = \sin t, \frac{dy}{dt} + x = \cos t$, Given that $x(0) = 2, y(0) = 0$. (CO 3) 10

7. Answer any one of the following:-

7-a. Evaluate 10

$\oint \vec{F} \cdot d\vec{r}$, by stokes theorem where $\vec{F} = y^2 \hat{i} + x^2 \hat{j} - (x+z) \hat{k}$ and C is the boundary of the triangle with vertices at $(0,0,0)$, $(1,0,0)$ and $(1,1,0)$. (CO4)

- 7-b. If $u = x + y + z$, $v = x^2 + y^2 + z^2$, $w = yz + zx + xy$ prove $\text{grad } u$, $\text{grad } v$ and $\text{grad } w$ are coplanar vectors. (CO4) 10

8. Answer any one of the following:-

- 8-a. (i) If Rs.7500 are borrowed at C.I at the rate of 4% per annum, find the amount to be paid after 2 years?(CO5) 10

(ii) What will be the ratio of SI earned by certain amount at same rate of interest for 6 years and that for 9 years? (CO5)

- 8-b. (i) Three friends invested Rs 700, Rs 600 and Rs 630 respectively. The first one invested for x months, second for (x+3) months and third for (x+6) months. What is the longest duration of investment, if ratio of share of first to third is 4 : 9?(CO5) 10

(ii) Suman and Chavi started a business by investing Rs 1960 and Rs 2450 respectively. Chavi got Rs 200 per month for her work. After 5 months, Suman added Rs 340 more and Chavi left. If after a year they get a total profit of Rs 18,850, then what total amount did Chavi get? (CO5)