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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

MCA

SEM: I - CARRY OVER THEORY EXAMINATION JUNE 2023

Subject: Discrete Mathematics

Time: 3 Hours

Max. Marks: 100

General Instructions:**IMP:** Verify that you have received the question paper with the correct course, code, branch etc.**1.** This Question paper comprises of **three Sections -A, B, & C.** It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.**2.** Maximum marks for each question are indicated on right -hand side of each question.**3.** Illustrate your answers with neat sketches wherever necessary.**4.** Assume suitable data if necessary.**5.** Preferably, write the answers in sequential order.**6.** No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.**SECTION A****20****1. Attempt all parts:-**

- 1-a. Power set of empty set has exactly _____ subset. (CO1) 1
- (a) One
(b) Two
(c) Zero
(d) Three
- 1-b. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = 3x - 4$. Then $f^{-1}(x)$ is given by: (CO1) 1
- (a) $(x + 4)/3$
(b) $(x/3) - 4$
(c) $3x + 4$
(d) None of these
- 1-c. The number of branches incident at the node of a graph is called? (CO2) 1
- (a) degree of the node
(b) order of the node
(c) status of the node

- (d) number of the node
- 1-d. If R is reflexive, symmetric and transitive then the relation is said to be: (CO2) 1
- (a) Binary Relation
 - (b) Equivalence Relation
 - (c) Irreflexive Relation
 - (d) Compatibility Relation
- 1-e. An algebraic structure _____ is called a semigroup. (CO3) 1
- (a) $(P, *)$
 - (b) $(Q, +, *)$
 - (c) $(P, +)$
 - (d) $(+, *)$
- 1-f. If $f(x) = y$ then $f^{-1}(y)$ is equal to _____. (CO3) 1
- (a) y
 - (b) x
 - (c) x^2
 - (d) none of the mentioned
- 1-g. Which of the following statement regarding sets is false? (CO4) 1
- (a) $A \cap A = A$
 - (b) $A \cup A = A$
 - (c) $A - (B \cap C) = (A - B) \cup (A - C)$
 - (d) $(A \cup B)' = A' \cup B'$
- 1-h. $\sim A \wedge B \rightarrow \sim(A \sqcap B)$ is a _____. (CO4) 1
- (a) Contingency
 - (b) tautology
 - (c) contradiction
 - (d) All the above
- 1-i. Which of these distributions is used for a testing hypothesis? (CO5) 1
- (a) Normal Distribution
 - (b) Chi- Squared Distribution
 - (c) Gamma Distribution
 - (d) Poisson Distribution
- 1-j. In how many ways 4 boys and 3 girls can be seated in a row so that they are alternate. (CO5) 1

- (a) 144
- (b) 288
- (c) 12
- (d) 256

2. Attempt all parts:-

- 2.a. Write the cardinal number of the set $Z = \{21, 28, 35, 42, 49\}$. (CO1) 2
- 2.b. Define Isomorphism of graphs. (CO2) 2
- 2.c. Define subset with an example. (CO3) 2
- 2.d. If P is false and Q is true then find the truth value of $(P \vee Q) \rightarrow \neg(P \wedge Q)$. (CO4) 2
- 2.e. Find the first Four terms each of the following recurrence relation: 2
 $T_n = 2T_{n-1} + 5T_{n-2}$ For all integers $n \geq 2$, $T_0 = 4$, $T_1 = 5$. (CO5)

SECTION B

30

3. Answer any five of the following:-

- 3-a. R and S are relation on $A = \{1, 2, 3\}$, $R = \{(1,1), (1,2), (2,3), (3,1), (3,3)\}$ and $S = \{(1,2), (1,3), (2,1), (3,3)\}$ then find RoS and SoR. (CO1) 6
- 3-b. Find the domain and range of the following real functions: (a) $y = x^2$, (b) $y = 3x - 7$. (CO1) 6
- 3-c. Give the analysis of insertion and deletion operations of nodes in binary search tree. (CO2) 6
- 3-d. Explain the Prim's algorithm to find minimal spanning tree for a graph. (CO2) 6
- 3.e. Find the number of four digit number possible with distinct digits. (CO3) 6
- 3.f. Justify whether the given equation is true or false: $\neg(p \rightarrow q) \equiv \neg p \rightarrow \neg q$. (CO4) 6
- 3.g. Solve the recurrence relation $a_r - 3a_{r-1} + 2a_{r-2} = 0$. (CO5) 6

SECTION C

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4. Answer any one of the following:-

- 4-a. Prove using mathematical induction that for all $n \geq 1$,
 $1 + 4 + 7 + \dots + (3n - 2) = n(3n - 1)/2$. (CO1) 10
- 4-b. Let the function $f: R \rightarrow R$ be defined by $f(x) = x^2$, then find out (i) $F^{-1}(9)$ (ii) $F^{-1}(-9)$. (CO1) 10

5. Answer any one of the following:-

- 5-a. Prove that the relation $\{(1,1), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2)\}$ on the set $\{1, 2, 3\}$ is neither reflective, nor irreflexive but transitive. (CO2) 10
- 5-b. Show that vertices of every planar graph can be properly coloured with five 10

colours. (CO2)

6. Answer any one of the following:-

- 6-a. Prove that the set of all rational numbers Q along with the operation of addition form a group. (CO3) 10
- 6-b. Let $f:R \rightarrow R$, $g:R \rightarrow R$, where R is the set of real numbers be given by $f(x) = x^2 - 2$ and $g(x) = x+4$ find $f \circ g$ and $g \circ f$. State whether these functions are bijective or not. (CO3) 10

7. Answer any one of the following:-

- 7-a. State the properties of tautology and contradiction relation and prove if $\sim A \wedge B \Rightarrow \sim(A \vee B)$ is a tautology or not. (CO4) 10
- 7-b. Construct the truth table for the given compound proposition:
 $(P \rightarrow (P \rightarrow Q)) \rightarrow ((Q \rightarrow R) \rightarrow (R \rightarrow S))$. (CO4) 10

8. Answer any one of the following:-

- 8-a. Determine the number of ways can the letters of the word 'DETAIL' be arranged in such a way that the vowels occupy only the odd positions. (CO5) 10
- 8-b. Prove that the generating function for series $1,1,1,1,1,\dots$ is $1/(1-x)$. (CO5) 10