Printed Page:- **Subject Code:- ACSBS0205**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: II - CARRY OVER THEORY EXAMINATION - MAY 2023

Subject: Linear Algebra

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* **three Sections -A, B, & C.** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION A **20**

1. Attempt all parts:-

1-a. $If\ A=\begin{bmatrix} a+ib & c+id \\ -c+id & a-ib \end{bmatrix}, where\ a^2+b^2+c^2+d^2=1, then\ A^{-1}\ is\ equal\ to$ 1

(CO1)

(a) $A=\begin{bmatrix} a+ib & -c+id \\ -c+id & a-ib \end{bmatrix}$

(b) $A=\begin{bmatrix} a-ib & -c-id \\ c+id & a+ib \end{bmatrix}$

(c) $A=\begin{bmatrix} a-ib & -c-id \\ c-id & a+ib \end{bmatrix}$

(d) none of these

1-b. *Cramer's rule fails for* 1

(CO1)

(a) $Determinant > 0$

(b) $Determinant < 0$

(c) $Determinant = 0$

(d) *none of these*

1-c. If the rank of a matrix A is 2, then the rank of A' is (CO2) 1

(a) 3

(b) 2

(c) 8

(d) none of these

1-d. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$ is (CO2) 1

(a) 2

(b) 3

(c) 4

(d) None of these

1-e. Which of the set of vectors are linearly dependent? (CO3) 1

(a) (1, 1, 4), (1, 0, 0), (1, 1, 0)

(b) (1, 2, 4), (1, 0, 0), (0, 1, 0), (0,0, 1)

(c) (1, 2, 4), (1, -1, 0), (0, 0, 1)

(d) None of these

1-f. Which statement is correct? (CO3) 1

(a) $\|\alpha\| \geq 0$ *if and only if* $\alpha = 0$

(b) $\|\alpha\| = 0$ *if and only if* $\alpha = 0$

(c) $\|\alpha\| = 0$ *if and only if* $\alpha = > 0$

(d) None of these

1-g. If A is a skew Hermitian matrix, then the principal diagonal elements of A are all (CO4) 1

(a) Real

(b) Negative

(c) Positive

(d) None of these

1-h. The eigen values of $4A^{-1} + 3A + 2I$, where $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ are (CO4) 1

(a) 1, 2

(b) 9, 15

(c) 3, 4

(d) None of these

1-i. 1

If 0 is a Eigen value of a matrix iff the matrix is (CO5)

(a) Non singular

(b) Unitary

(c) Singular

(d) None of these

1-j. If $A=\begin{bmatrix}3 & 0\\ 4 & 5\end{bmatrix}$ then the Eigen value of A^TA are (CO5) 1

(a) 45 ,5

(b) 45, 45

(c) 5 , 5

(d) None of these

2. Attempt all parts:-

2.a. If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, show that $A^2-5A=2I$, where I is the unit matrix of order 2.(CO1) 2

2.b. Solve the following system of equations x + y + z = 0, x + 2y - z = 0, 2x + y + 3z = 0. (CO2) 2

2.c. Show that the vectors {(1,1,0,0), (0,1,-1,0), (0,0,0,3)} in R^4 are linearly independent. (CO3) 2

2.d. For the matrix A, Find the sum of eigen values where $A=\begin{bmatrix}-2 & 1 & 0\\ 2 & 1 & 1\\ 0 & 1 & 0\end{bmatrix}$. (CO4) 2

2.e. In singular value decomposition if $A=\begin{bmatrix}2 & 2\\ 1 & 1\end{bmatrix}$ then find S ? (CO5) 2

SECTION B **30**

3. Answer any <u>five</u> of the following:-

3-a. Find the inverse of the matrix $\begin{bmatrix}2 & 3 & 4\\ 4 & 3 & 1\\ 1 & 2 & 4\end{bmatrix}$ by elementary transformation.(CO1) 6

3-b. Reduce the matrix $A=\begin{bmatrix}-1 & 2 & -2\\ 1 & 2 & 1\\ -1 & -1 & 0\end{bmatrix}$ to upper traingular form.(CO1) 6

3-c. Find all values of μ for which rank of the matrix $A=\begin{bmatrix}\mu & -1 & 0 & 0\\ 0 & \mu & -1 & 0\\ 0 & 0 & \mu & -1\\ -6 & 11 & -6 & 1\end{bmatrix}$ is equal to 3. (CO2) 6

3-d. Show that the vectors X_1 = [1,2,4], X_2 = [2,-1,3], X_3 = [0,1,2] and X_4 = [-3,7,2] are linearly dependent and find the relation between them. (CO2) 6

3.e. If the vectors (0,1,a), (1,a,1), (a,1,0) of vector space $R^3(R)$ are L.D., find the value of a. (CO3) 6

3.f. Express the matrix $\begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. (CO4) 6

3.g. Given the following data, Using PCA find the covariance.(CO5) 6

x:	4	8	13	7
y:	11	4	5	14

SECTION C **50**

4. Answer any <u>one</u> of the following:-

4-a. Solve the system of equations by matrix method: (CO1) 10
x+2y-3z=4, 2x+3y+2z=2 and 3x-3y-4z=11.

4-b. If $A = \begin{bmatrix} 1 & 2 & 1 \\ a & 0 & 4 \\ 1 & 1 & 1 \end{bmatrix}$ and adj(adj. A) = A, find a. (CO1) 10

5. Answer any <u>one</u> of the following:-

5-a. Determine the value of λ and μ so that the equations x + y + z = 6, x + 2y + 3z = 10, x + 2y + λz = μ. (CO2) 10
(i) No solution
(ii) Unique solution
(iii) Infinite solution

5-b. Show that the following system of equation 3x+4y+5z = a, 4x+5y+6z = b, 5x+6y+7z = c is consistent only if a , b and c are in arithmetic progression (AP). (CO2) 10

6. Answer any <u>one</u> of the following:-

6-a. If W_1 and W_2 are subspaces of the vector space $R^4(R)$ generated by S_1={(1,1,0,-1), (1,2,3,0), (2,3,3,-1)}, S_2={(1,2,2,-2), (2,3,2,-3), (1,3,4,-3)} respectively, Determine-(CO3) 10
(a) dim(W_1 + W_2)
(b) dim(W_1 ∩ W_2)

6-b. Apply Gram-schmidt process to the vectors α_1=(1,0,1), α_2=(1,0,-1), α_3=(0,3,4) to obtain the orthonormal basis for $V_3(R)$.(CO3) 10

7. Answer any <u>one</u> of the following:-

7-a. Show that the mapping $T: V_3(R) \to V_2(R)$ defined as $T(a,b,c) = (a,b)$ is a linear transformation. (CO4) 10

7-b. Find the eigen values of $3A^3 + 5A^2 - 6A + 2I$ where 10

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}.$$ (CO4)

8. Answer any <u>one</u> of the following:-

8-a. Find a singular value decomposition of $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.(CO5) 10

8-b. Given the following data, use PCA to reduce the dimension from 2 to 1.(CO5) 10

Feature	Example 1	Example 2	Example 3	Example 4
x:	4	8	13	7
y:	11	4	5	14