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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: IV CARRY OVER THEORY EXAMINATION - APRIL 2023

Subject: Engineering Mathematics-III

Time: 3 Hours

Max. Marks: 100

General Instructions:

IMP: Verify that you have received the question paper with the correct course, code, branch etc.

1. This Question paper comprises of **three Sections -A, B, & C.** It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.

2. Maximum marks for each question are indicated on right -hand side of each question.

3. Illustrate your answers with neat sketches wherever necessary.

4. Assume suitable data if necessary.

5. Preferably, write the answers in sequential order.

6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A

20

1. Attempt all parts:-

1-a. The bilinear transformation is (CO1)

1

(a) $f(z) = \frac{az+b}{cz+d}, ad-bc=0$

(b) $f(z) = \frac{az+b}{cz+d}, ad-bc \neq 0$

(c) $f(z) = \frac{az+b}{cz+d}, ad-bc=1$

(d) None of these

1-b. Analytic function is also known as (CO1)

1

(a) Regular function

(b) Orthogonal Transformation

(c) Affine Function

(d) None of these

1-c. Value of $\int_C \frac{dz}{z(z+\pi i)}$, where C is a curve $|z+3i|=1$ is (CO2)

1

(a) -1

- (b) -2
- (c) 1
- (d) None of these

1-d. Residue of the function $f(z) = \frac{1}{z^2(z-i)}$ at $z = i$ is (CO2) 1

- (a) 2π
- (b) -2π
- (c) -1
- (d) 1

1-e. Classification of partial differential equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y}$ is (CO3) 1

- (a) Parabola
- (b) Hyperbola
- (c) Elliptic
- (d) None of these

1-f. For a partial differential equation, $A u_{xx} + B u_{xy} + C u_{yy} = 0$ if $B^2 - 4AC = 0$ then equation is called (CO3) 1

- (a) Hyperbola
- (b) Elliptic
- (c) Parabola
- (d) None of these

1-g. The Fourier cosine transform of e^{-x} , $x \geq 0$ is (CO4) 1

- (a) $\frac{p}{1+p^2}$
- (b) $\frac{1}{1+p^2}$
- (c) $\frac{p}{1-p^2}$
- (d) $\frac{1}{1-p^2}$

1-h. Z-Transform of $f(k) = \begin{cases} 0, & k > 0 \\ 1, & k \leq 0 \end{cases}$ is (CO4) 1

- (a) $\frac{1}{2-z}$
- (b) $\frac{1}{1-z}$

$$\frac{1}{2+z}$$

(c) $2+z$

(d) None of these

1-i. 12 buckets of water fill a tank when the capacity of each bucket is 13.5 litres. How many buckets will be needed to fill the same tank, if the capacity of each bucket is 9 litres? (CO5) 1

(a) 8

(b) 15

(c) 16

(d) 18

1-j. A can finish a work in 18 days and B can do the same work in half the time taken by A. Then, working together, what part of the same work they can finish in a day? (CO5) 1

(a) $1/6$

(b) $1/9$

(c) $2/5$

(d) $2/7$

2. Attempt all parts:-

2.a. Write a short note on harmonic function. (CO1) 2

2.b. Evaluate: $\oint_C \frac{e^{-z}}{z+1} dz$, $C: |z| = 2$. (CO2) 2

2.c. Write the solution of 1-d wave equation. (CO3) 2

2.d. Find z -Transform of $\{a^k\}$, $k \geq 0$. (CO4) 2

2.e. A man can row upstream at 10 mph and downstream at 18 kmph. Find the man's rate in still water? (CO5) 2

SECTION B

30

3. Answer any five of the following:-

3-a. Find the value of p such that the function $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \frac{px}{y}$ is analytic. Also find $f'(z)$? (CO1) 6

3-b. Discuss the analyticity of the function $f(z) = z\bar{z}$? (CO1) 6

3-c. Evaluate $\oint_C \frac{e^z}{z^2 + 1} dz$; $C \equiv |z + i| = 1$. (CO2) 6

3-d. Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z+1)(z+2)} dz$; $C \equiv |z| = 3$. (CO2) 6

- 3.e. Use the method of separation of variables to solve the equation $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$, subject to the condition is $u(x, 0) = 6e^{-3x}$. (CO3) 6
- 3.f. Find the Fourier transform of $F(t) = \begin{cases} t, & \text{for } |t| < a \\ 0, & \text{for } |t| > a \end{cases}$. (CO4) 6
- 3.g. To complete a piece of work, Samir takes 6 days and Tanvir takes 8 days alone respectively. Samir and Tanvir took Rs. 2400 to do this work. When Amir joined them, the work was done in 3 days. What amount was paid to Amir? (CO5) 6

SECTION C

50

4. Answer any one of the following:-

- 4-a. Find the bilinear transformation which maps the points $z = 0, -1, i$ into the points $w = i, 0, \infty$. Hence find the image of $z = 2$. (CO1) 10
- 4-b. If $f(z) = u + iv$ is an analytic function of $z = x + iy$ and $u - v = e^{-x}[(x - y)\sin y - (x + y)\cos y]$ then construct $f(z)$ in terms of z . (CO1) 10

5. Answer any one of the following:-

- 5-a. Evaluate $\int_0^{2\pi} \frac{1}{3 - 2 \cos \theta + \sin \theta} d\theta$ using contour integration. (CO2) 10
- 5-b. Determine the poles of the following function and residues at each poles: $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and hence evaluate $\int_C f(z) dz$ where C is the circle $|z| = 3$. (CO2) 10

6. Answer any one of the following:-

- 6-a. Solve the PDE: $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$. (CO3) 10
- 6-b. Solve the PDE: $r + 2s + t = 2(y - x) + \sin(x - y)$. (CO3) 10

7. Answer any one of the following:-

- 7-a. Solve by z -transform: $y_{k+1} - 2y_{k-1} = 0, k \geq 1, y(0) = 1$. (CO4) 10
- 7-b. Find the Fourier cosine transform of the following functions: 10

$$F(x) = \begin{cases} x, & 0 < x < \frac{1}{2} \\ 1 - x, & \frac{1}{2} < x < 1 \\ 0, & x > 1 \end{cases} \quad (\text{CO4})$$

8. Answer any one of the following:-

- 8-a. **(i)** A clock is set right at 8 a.m. The clock gains 10 minutes in 24 hours. What 10
will be the true time when the clock indicates 1 p.m. on the following day?
(ii) What was the day of the week on 16th July, 1776? (CO5)
- 8-b. **(i)** A car takes 15 minutes less to cover a distance of 75 km, if it increases its 10
speed by 10 km/hr from its usual speed. How much time would it take to cover
a distance of 300 km using this speed?
(ii) Two men starting from the same place walk at the rate of 5 kmph and 5.5
kmph respectively. What time will they take to be 8.5 km apart, if they walk in
the same direction? (CO5)

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