

# NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA 

 (An Autonomous Institute Affiliated to AKTU, Lucknow) B.TechSEM: III - CARRY OVER THEORY EXAMINATION - APRIL 2023
Subject: Engineering Mathematics-III
Time: 3 Hours
Max. Marks: 100

## General Instructions:

IMP: Verify that you have received the question paper with the correct course, code, branch etc.

1. This Question paper comprises of three Sections -A, B, \& C. It consists of Multiple Choice Questions (MCQ's) \& Subjective type questions.
2. Maximum marks for each question are indicated on right -hand side of each question.
3. Illustrate your answers with neat sketches wherever necessary.
4. Assume suitable data if necessary.
5. Preferably, write the answers in sequential order.
6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

## SECTION A

## 1. Attempt all parts:-

1-a. Let $\mathbf{u}(\mathrm{x}, \mathrm{y})$ be a harmonic function and $\mathbf{v}(\mathbf{x}, \mathrm{y})$ is a harmonic conjugate of $\mathbf{u}(\mathrm{x}, \mathrm{y})$ then, which one is true: (CO1)
(a) $\mathbf{v}$ is harmonic and $u \& v$ satisfy C-R equations.
(b) $v$ is not harmonic and $u \& v$ satisfy C-R equations.
(c) $v$ is harmonic and $\mathbf{u} \& v$ do not satisfy C-R equations.
(d) $v$ is not harmonic and $u \& v$ do not satisfy C-R equations.

1-b. Which of the following function is unbounded? (CO1)
(a) $\operatorname{Sin} z$
(b) $\operatorname{Cos} z$
(c) $e^{z}$
(d) All of the above

1-c. $f(z)=\sin \left(\frac{1}{z-a}\right)$ is (CO2)
At $z=a$, nature of singularity of
(a) Removable
(b) Isolated essential
(c) Non isolated Essential
(d) Pole

1-d. The region of validity for Taylor's series about $z=0$ of the function $e^{z}$ is (CO2)
(a) $|z|<1$
(b) $|z|>1$
(c) $|z|<\infty$
(d) None of these

1-e. Classify the PDE $\mathbf{u}_{\mathrm{zx}}-\mathrm{u}_{\mathrm{yy}}=0$ (CO3)
(a) Parabolic
(b) Elliptic
(c) Hyperbolic
(d) None of these

1-f. The one dimensional heat equation can be solved using separable method. The constant which is appears in the solution should be (CO3)
(a) Positive
(b) Negative
(c) Zero
(d) None of these

1-g.
Value of $\int_{4}^{5.2} \log x d x$ using Simpson's one third rule with interval size 0.3 is (CO4)
(a) 1.60
(b) 1.51
(c) 1.06
(d) 1.83

1-h. The process of finding the values inside the interval ( $x_{0}, x_{1}$ ) is called (CO4)
(a) Interpolation
(b) Extrapolation
(c) Iterative
(d) Polynomial Equation

1-i. A train can travel 50\% faster than a car. Both start from point $A$ at the same time and reach point B 75 kms away from A at the same time. On the way, however, the train lost about 12.5 minutes while stopping at the stations. The speed of the car is: (CO5)
(a) 100 kmph
(b) 110 kmph
(c) 120 kmph
(d) None of these

1-j. Raj swims 26 km downstream in same time as 14 km upstream. What is his speed in still water if speed of stream is $3 \mathrm{~km} / \mathrm{hr}$ ? (CO5)
(a) $10 \mathrm{~km} / \mathrm{hr}$
(b) $12 \mathrm{~km} / \mathrm{hr}$
(c) $7 \mathrm{~km} / \mathrm{hr}$
(d) None of these

## 2. Attempt all parts:-

2.a. Write Cauchy Riemann equations. (CO1).
2.b. $\quad$ Expand $\frac{1}{(1+z)(z+3)}$ in the region $|z|<1$. ${ }_{\text {(CO2) }}$
2.c. Find the C.F. of $\left(D-D^{\prime}-1\right) z=x^{2} y \cdot(C O 3)$
2.d. Write Regula false method. (CO4)
2.e. $A, P, R, X, S$ and $Z$ are sitting in a row. $S$ and $Z$ are in the Centre. $A$ and $P$ are at the ends. $R$ is sitting to the left of $A$. Who is to the right of $P$ ? (CO5)

## SECTION B

## 3. Answer any five of the following:-

3-a. Prove that the function $f(z)=\cosh z$ is analytic in entire complex plane. (CO1)
3-b. Determine an analytic function $f(z)$ in terms of $z$ whose real part is $e^{-x}(x \cos y+y \sin y) \cdot f(0)=1$. (CO1)
3-c. Evaluate $\oint_{C} \frac{z^{2}+5}{(z-3)} d z$; Where $C$ is a circle $|z|=4$.
3-d. Evaluate $\int_{0}^{2 \pi} \frac{\cos 2 \theta}{5+4 \cos \theta} d \theta$. (CO2)
3.e. Solve the PDE: $\left(D^{2}+7 D^{\prime}+12 D^{\prime 2}\right) z=\operatorname{sinx}(C O 3)$.
3.f. Find the cubic polynomial which takes the following values:

| $x$ | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- |
| $y$ | 6 | 7 | 8 | 15 |

find $y$ (0.5). (CO4)
3.g. A man takes 2.2 times as long to row a distance upstream as to row the same distance downstream. If he can row 55 km downstream in 2 hours 30 minutes,
what is the speed of the boat in still water? (CO5)

## SECTION C

## 4. Answer any one of the following:-

4-a. If function $f(z)=\left\{\frac{\left(\overline{)^{2}}\right.}{z}, \quad z \neq 0\right\}, f(0)=0$
then Check that C-R equations are satisfied at origin or not. Does $f^{\prime}(0)$ exist? (CO1)

4-b. Show that the transformation ${ }^{w=\frac{i z+2}{4 z+i}}$ maps the real axis in the $z$-plane in to a circle in the w-plane.
Find the center and the radius of the circle and the point in the z-plane which is mapped on the center of the circle. (CO1)

## 5. Answer any one of the following:-

5-a. $\quad$ Expand $f(z)=\frac{1}{z^{2}-3 z+2}$
(i) $\mid$ z| $<1$
(ii) $1<|z|<2$
(iii) $0<|z-1|<1$
(iv) $|z|>2$
(CO2)
5-b. Determine the poles of the following function and residues at each poles: $f(z)=\frac{z-1}{(z+1)^{2}(z-2)}$ and
hence evaluate $\int_{C} f(z) d z$ where $C$ is the circle $|z-i|=2$.

## 6. Answer any one of the following:-

6-a. Solve the PDE: $s+p-q=z+x y$.(CO3) 10
6-b. $\quad$ Solve the PDE: $\left(D^{2}-D^{\prime}+D^{\prime}-1\right) z=\cos (x+2 y)+e^{y} .(C O 3)$
7. Answer any one of the following:-

7-a. Solve the following system of linear equations using Gauss Seidel method:
$10 x+2 y+z=9 ; 2 x+20 y-2 z=-44 ;-2 x+3 y+10 z=22$. (CO4)
7-b. Apply Crouts method to solve the equations 10
$3 x+2 y+7 z=4 ; 2 x+3 y+z=5 ; 3 x+4 y+z=7$ (CO4)

## 8. Answer any one of the following:-

8-a. (i) A boat, while going downstream in a river covered a distance of 50 miles at 10 an average speed of 60 miles per hour. While returning, because of the water resistance, it took 1 hour 15 minutes to cover the same distance. What was the average speed during the whole journey?
(ii) A man can row 40 km upstream and 55 km downstream in 13 hours. Also,
he can row 30 km upstream and 44 km downstream in 10 hours. Find the speed of the man in still water and the speed of the current? (CO5)

8-b. (i) How many rotations will the hour hand of a clock complete in 72 hours? 10
(ii) Through what angle does the minute hand of a clock turn in 5 minutes? (CO5)

