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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: I - CARRY OVER THEORY EXAMINATION - MAY 2023

Subject: Mathematical Foundations – II

Time: 3 Hours

Max. Marks: 100

General Instructions:

IMP: Verify that you have received the question paper with the correct course, code, branch etc.

1. This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.
2. Maximum marks for each question are indicated on right -hand side of each question.
3. Illustrate your answers with neat sketches wherever necessary.
4. Assume suitable data if necessary.
5. Preferably, write the answers in sequential order.
6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A

20

1. Attempt all parts:-

- 1-a. Integral value of $\int_0^{\pi/2} 128 \sin^2 \theta \cos^{10} \theta d\theta$ (CO1) 1
- (a) 0
- (b) $21\pi/16$
- (c) $21\pi/4$
- (d) None of these
- 1-b. The improper integral $\int_1^{\infty} \frac{dx}{x^{3/2}}$ (CO1) 1
- (a) Divergent
- (b) Converges to 2
- (c) Converges to 1
- (d) None of these
- 1-c. Find the roots of the of auxiliary equation the differential equation 1
- $(D^3 + 2D^2 - D - 2)y = e^x$ CO 2

(a) 1,1, -2

(b) 1,-1, -2

(c) 1,-1, 2

(d) 1,-1, -3

1-d. Which of the following is a solution of $(D^2 - 6D + 9)y = 0$ CO 2 1

(a) e^{2x}

(b) e^{-3x}

(c) $e^{2x} + e^{-3x} + 1$

(d) None of these

1-e. The linear partial differential equation $2\frac{\partial^2 u}{\partial t^2} + 4\frac{\partial^2 u}{\partial x \partial t} + 3\frac{\partial^2 u}{\partial x^2} = 0$ is (CO3) 1

(a) Parabolic

(b) Elliptic

(c) Hyperbolic

(d) None of these

1-f. The linear partial differential equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ is (CO3) 1

(a) Parabolic

(b) Elliptic

(c) Hyperbolic

(d) None of these

1-g. Laplace transform of the function $F(t) = \sinh 3t$ is CO 4 1

(a) $\frac{3}{s^2 - 9}$

(b) $\frac{3}{s^2 + 9}$

(c) $\frac{1}{s^2 + 9}$

(d) $\frac{s}{s^2 + 9}$

1-h. Laplace Transform of $\cos t$ is CO 4 1

(a) $\frac{s}{s^2 - 1}$

(b) $\frac{s}{s^2 + 1}$

(c) $\frac{1}{s^2 - 1}$

(d) $\frac{-s}{s^2 - 1}$

- 1-i. A shopkeeper bought 30 kg of rice at the rate of Rs. 70 per kg and 20 kg of rice at the rate of Rs. 70.75 per kg. If he mixed the two brands of rice and sold the mixture at Rs. 80.50 per kg, his gain is (CO5) 1
- (a) Rs. 450
- (b) Rs. 510
- (c) Rs. 525
- (d) Rs. 485
- 1-j. 729 ml of a mixture contains milk and water in the ratio 7:2. How much more water is to be added to get a new mixture containing milk and water in the ratio 7:3? (CO5) 1
- (a) 81 ml
- (b) 60ml
- (c) 71 ml
- (d) 52ml

2. Attempt all parts:-

- 2.a. Test the convergence of integral $\int_3^{\infty} \frac{dx}{(x-2)^2}$. (CO1) 2
- 2.b. Write the relation between P and Q if x is a part of the C F of the linear differential equation $d^2y/dx^2 + P(dy/dx) + Qy = R$ (CO 2) 2
- 2.c. Solve the partial differential equation $yzp + zxq = xy$ (CO3) 2
- 2.d. Write the statement of First shifting property of Laplace transform. (CO 4) 2
- 2.e. ₹ 385 were divided among P, Q and R in such a way that P had ₹ 20 more than Q and R had ₹ 15 more than P. How much was R's share? (CO5) 2

SECTION B

30

3. Answer any five of the following:-

- 3-a. Evaluate $\iint_D 2yx^2 + 9y^3 dx dy$, where D is the region bounded by $y = \frac{2}{3}x$ and $y = 2\sqrt{x}$ (CO1) 6
- 3-b. Apply Dirichlet's integral to evaluate $\iiint \int dx dy dz$, where $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (CO1) 6
- 3-c. Solve the differential equation $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$ (CO2) 6
- 3-d. Solve $\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + y = x^2 e^{-x} \cos x$. (CO2) 6
- 3.e. Solve the linear partial differential equation $(x^2 D^2 - y^2 D'^2)z = x^2 y$. (CO3) 6
- 3.f. Find the Laplace Transform of the function $F(t) = \sin^3 t \cdot e^{-2t}$ (CO 4). 6

- 3.g. (i) A sum of ₹ 350 made up of 110 coins, which are of either ₹ 1 or ₹ 5 denominations. How many coins are of ₹ 5? 6
(ii) $a = 2b = 3c = 4d$, find the $a : b : c : d$? (CO5)

SECTION C

50

4. Answer any one of the following:-

- 4-a. Change the order of integration: $\int_0^a \int_0^{2\sqrt{ay}} f(x, y) dx dy + \int_0^{3a} \int_0^{3a-y} f(x, y) dx dy$ (CO1) 10

- 4-b. Evaluate $\int \int (x^2 + y^2)^{7/2} dx dy$ over the circle $x^2 + y^2 = 1$ (CO1) 10

5. Answer any one of the following:-

- 5-a. Solve the differential equations by method of variation of parameters : $(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$. (CO2) 10

- 5-b. Solve the differential equation : $\frac{d^2y}{dx^2} - y = x \sin 3x + \cos x$. (CO2) 10

6. Answer any one of the following:-

- 6-a. Solve : $(x^2 D^2 + 2xy DD' + y^2 D'^2)z = x^m y^n$. (CO3) 10

- 6-b. Solve : $x^2 r + 2y^2 t + px - 3xys + 2qy = x + 2y$. (CO3) 10

7. Answer any one of the following:-

- 7-a. Find Laplace transform of the function $f(t) = \begin{cases} 1 & 0 \leq t < 2 \\ -1 & 2 \leq t < 4 \end{cases}$ Where $f(t+4) = f(t)$ (CO4) 10

- 7-b. State Convolution theorem and hence find the inverse laplace transform of $\frac{s}{(s^2 + a^2)^2}$. (CO4) 10

8. Answer any one of the following:-

- 8-a. (i) Abhay's age after 6 year will be three-seventh of his father's age. Ten years ago, the ratio of their ages was 1:5. What is Abhay's father's age at present? 10
(ii) Sumit, Ravi and Puneet invest ₹ 45000, ₹ 81000 and ₹ 90000 respectively to start a business. At the end of the year the total profit is ₹ 4800. 30% of the total profit gives in charity and rest is divided among them. What will be the share of Sumit? (CO5)

- 8-b. (i) Amit started a business by investing ₹ 30,000. Rahul joined the business after some time and invested ₹ 20,000. At the end of the year, profit was divided in the ratio of 2: 1. After how many months did Rahul join the business? 10

- (ii) The monthly income of Komal and Asha are in the ratio of 4: 3. Their monthly expenses are in the ratio of 3: 2. However both saves ₹ 600 per month. What is their total monthly income? (CO5)