Printed Page:- **Subject Code:- AAS0104**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: I - CARRY OVER THEORY EXAMINATION - APRIL 2023

Subject: Mathematical Foundations-I

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* **three Sections -A, B, & C.** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION A **20**

1. Attempt all parts:-

1-a. If the eigen values of a matrix A are 4, 5, 7 then write the eigen values of A^{-1} are. (CO1) 1

(a) 4, 5^2, 7^3

(b) 4,5,7

(c) 1/4, 1/5, 1/7

(d) none of these

1-b. If every minor of order r of a matrix A is zero, then the rank A is (CO1) 1

(a) greater than r

(b) equal to r

(c) less than or equal to r

(d) less than r

1-c. Which condition exist for function to be linear transformation ? (CO2) 1

(a) $T(a\alpha + b\beta) = aT(\beta) + bT(\alpha)$

(b) $T(a\alpha + b\beta) = aT(\alpha) + bT(\alpha)$

(c) $T(a\alpha + b\beta) = aT(\alpha) + bT(\beta)$

(d) None of these

1-d. If T be a linear transformation from U into V , then according to rank and nullity theorem: (CO2) 1

(a) Rank(T) - Nullity(T) = Dim U

(b) Rank(T) + Nullity(T) = Dim U

(c) Rank(T) + Nullity(T) = Dim V

(d) None of these

1-e. Asymptotes parallel to y-axis of the curve $y^2(a+x) = x^2(3a-x)$ (CO3) 1

(a) $x = a$

(b) $x = -a$

(c) $x = 3a$

(d) $x = -3a$

1-f. The nth derivative of cos(ax+b) is (CO 3) 1

(a) $a^n \cos(ax+b)$

(b) $a^n \cos\left(ax+b+\frac{n\pi}{2}\right)$

(c) $a^n \cos\left(ax+b+\frac{n\pi}{4}\right)$

(d) None of these

1-g. Percentage error in the area of a rectangle when an error of +1 percent is made in measuring its length and breadth is given by (CO4) 1

(a) 4%

(b) 5%

(c) 2%

(d) 6%

1-h. If $u = x(1-y), v = xy$ then the value of the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ is (CO4) 1

(a) 7x

(b) 3x

(c) x

(d) 4x

1-i. If out of 10 selected students for an examination, 3 were of 20 years, age, 4 of 21 and 3 of 22 years, the average age of the group is (CO5) 1

(a) 22 years

(b) 21 years

(c) 21.5 years

(d) 20 years

1-j. Missing terms in the series 0, 7, 26, 63, 124, --- is (CO5) 1

(a) 210

(b) 215

(c) 211

(d) 224

2. Attempt all parts:-

2.a. For what value of '*x*', the eigen values of of the given matrix A are real 2

$A = \begin{bmatrix} 10 & 5+i & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$.. (CO1)

2.b. Show that the three vectors (1, 1, -1), (2, -3, 5) and (-2, 1, 4) of R^3 are linearly independent. (CO2) 2

2.c. Find the nth derivative of $y = \frac{1}{(2x+3)(3x-1)}$. (CO3) 2

2.d. Find the stationary points of $f(x,y) = 5x^2 + 10y^2 + 12xy - 4x - 6y + 1$. (CO4) 2

2.e. If in a certain code "RANGE" is coded as 12345 and "RANDOM" is coded as 123678. Then find the code for the word "MANGO". (CO5) 2

SECTION B **30**

3. Answer any <u>five</u> of the following:-

3-a. Find the rank of matrix by reducing it to normal form $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 1 & 2 & -8 \end{bmatrix}$ (CO1) 6

3-b. Test the consistency of system of equation $10y + 3z = 0,\ 3x + 3y + z = 0,\ 2x - 3y - z = 5,\ x + 2y = 4$. (CO1) 6

3-c. Show that the vectors (0, 1, 1), (1, 0, 1) and (1, 1, 0) form a basis of R^3. (CO2) 6

3-d. Prove that the set S = { (1, 3, -1), 2, 7, -3), (4, 8, -7) } spans R^3 . (CO2) 6

3.e. If $\mathbf{u} = \tan^{-1}\left(\frac{\mathbf{x}^3 + \mathbf{y}^3}{\mathbf{x} + \mathbf{y}}\right)$ then find the value of $\mathbf{x}\frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \mathbf{y}\frac{\partial \mathbf{u}}{\partial \mathbf{y}}$. (CO3) 6

3.f. Examine the function $f(x,y) = y^2 + 4xy + 3x^2 + x^3$ for extreme values. (CO4) 6

3.g. Water tax is increased by 20 % but its consumption is decreased by 20 %. Then find the increase or decrease in the expenditure of the money. (CO5) 6

SECTION C 50

4. Answer any <u>one</u> of the following:-

4-a. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. (CO1) 10

4-b. Verify Caley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence compute A^{-1}. Also evaluate $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$.. (CO1) 10

5. Answer any <u>one</u> of the following:-

5-a. Show that the mapping $T: V_2(R) \to V_3(R)$ *defined as* $T(a, b) = (a + b, a - b, b)$ is a linear transformation from $V_2(R)$ into $V_3(R)$. Find the range, rank, null-space and nullity of T. (CO2) 10

5-b. Define inner product space. Then show that $u = (u_1, u_2), v = (v_1, v_2)$ *in* R^2 *defined by* $\langle u, v \rangle = 4u_1v_1 + 5u_2v_2$ is inner product space. (CO2) 10

6. Answer any <u>one</u> of the following:-

6-a. If $u = f(r)$, where $r = \sqrt{x^2 + y^2}$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$. (CO3) 10

6-b. If $y = e^{a \sin^{-1} x}$, then prove that $(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - (n^2 + a^2) y_n = 0$ (CO3) 10

7. Answer any <u>one</u> of the following:-

7-a. If u, v, w are the roots of the cubic equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (CO4) 10

7-b. Expand $e^x \cos y$ in the powers of $(x - 1)$ and $\left(y - \frac{\pi}{4}\right)$ upto the third degree terms. (CO4) 10

8. Answer any <u>one</u> of the following:-

8-a. (i) If the price of an item is decreased by 10% and then increased by 10%, then find the net effect on the price of the item. (CO5) 10

(ii) The average marks obtained by 40 students of a class is 86. If the 5 highest marks are removed, the average reduced by one marks. Find the average marks of the top 5 students. (CO5)

(iii) Find the missing terms: 1, 2, 6, 7, 21, 22, 66, 67, ? (CO5)

8-b. (i) Pankaj purchased an item for Rs. 7500 and sold it at the gain of 24%. From that amount he purchased another item and sold it at the loss of 20%. What is his overall gain/loss? (CO5) 10

(ii) The average of runs of a cricket player of 20 innings was 32. How many runs must he make in his next innings so as to increase his average of runs by 4 ? (CO5)

(iii) In certain code language, ROCK=47 and LATE=38. Find the code for FOOL. (CO5)