Printed Page:- **Subject Code:- AAS0103** /AASH0103

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: I - CARRY OVER THEORY EXAMINATION - MAY 2023

Subject: Engineering mathematics I

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* **three Sections -A, B, & C.** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION A **20**

1. Attempt all parts:-

1-a. If A is matrix such that there exists a square submatrix of order r which is non-singular and every submatrix of order r+1 or more is singular, then (CO1) 1

(a) rank A = r+1

(b) rank A = r

(c) rank A > r

(d) $rank\ A \geq r+1$

1-b. The largest eigen value of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 5 \\ 0 & 0 & 4 \end{bmatrix}$ is (CO1) 1

(a) 1

(b) 5

(c) 4

(d) None of these

1-c. The nth derivative of log (ax+b) is (CO2) 1

(a) $\dfrac{(-1)^n n! a^n}{(ax+b)^n}$

(b) $\dfrac{(-1)^{n-1} n!\, a^n}{(ax+b)^{n+1}}$

(c) $\dfrac{(-1)^n (n-1)!\, a^n}{(ax+b)^n}$

(d) $\dfrac{(-1)^{n-1} (n-1)!\, a^n}{(ax+b)^n}$

1-d. If $z = xy\, f\left(\dfrac{x}{y}\right)$ then the value of $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial x}$ is (CO2) 1

(a) 0

(b) z

(c) 2z

(d) 3z

1-e. If δx is error in x, then relative error of x is given by n (CO3) 1

(a) $\left|\dfrac{1}{\delta x}\right|$

(b) $\left|\dfrac{x}{\delta x}\right|$

(c) $\left|\dfrac{\delta x}{x}\right|$

(d) $x.\left|\dfrac{\delta x}{x}\right|$

1-f. With usual notation a function f(x, y) has a saddle point at (a, b) if (CO3) 1

(a) $rt - s^2 < 0$

(b) $rt - s^2 > 0$

(c) $rt - s^2 = 0$

(d) $rt = s$

1-g. The value of $\int_0^{\pi/2}\int_0^{\pi/2} \sin(x+y)\,dx\,dy$ is (CO 4) 1

(a) 0

(b) π

(c) $\pi/2$

(d) 2

1-h. If u= x+y and v= x-2y ,then the area element dxdy is replaced by (CO4) 1

(a) $\dfrac{1}{2} dudv$

(b) $\frac{1}{3}dudv$

(c) $dudv$

(d) None of these

1-i. A got 37.5 % marks less than B, then by what percent the marks of B is more than the marks of A? (CO5) 1

(a) 60%

(b) 37.5%

(c) 27.27%

(d) 40%

1-j. Reduction in price of sugar by 20% allows a household to buy 45 kg more for Rs.450. Find the original price of the sugar. (CO5) 1

(a) Rs 2 Kg

(b) Rs 2.5 Kg

(c) Rs 3 Kg

(d) None

2. Attempt all parts:-

2.a. For what value of '*x*', the eigen values of of the given matrix A are real $A = \begin{bmatrix} 10 & 5+i & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$. (CO1) 2

2.b. If $u = \log\left(\frac{x^4+y^4}{x+y}\right)$ Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3$ (CO2) 2

2.c. What is the maximum value of the function $f(x,y) = 1-x^2-y^2$. (CO3) 2

2.d. Find the area of region bounded by the parabola $x=y^2$ and the line $y=x$. (CO 4) 2

2.e. A single discount equivalent to three successive discounts of 5%, 10%, 20% is? (CO5) 2

SECTION B **30**

3. Answer any five of the following:-

3-a. Check the given system of linear equation is $x-y+2z=2,\ 2x+y+4z=7,\ 4x-y+z=4$. consistent or inconsistent. (CO1) 6

3-b. Find the rank of matrix $\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 1 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$ by reducing it to Echelon form. (CO1) 6

3-c. If $x^x y^y z^z = c$ then Show that at $x = y = z$, $\left(\frac{\partial^2 z}{\partial x \partial y}\right)_{x=y=z} = (x \log_e x)^{-1}$, (CO2) 6

3-d. Find the n th differential coefficients of $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$. (CO2) 6

3.e. Compute an approximate value of $[(0.98)^2 + (2.01)^2 + (1.94)^2]^{1/2}$. (CO 3) 6

3.f. Evaluate $\iint_R \sqrt{x^2+y^2}\, dx\, dy$ by changing into polar coordinate, where R is bounded by $x^2+y^2=4$ and $x^2+y^2=9$. (CO 4) 6

3.g. The average of 50 numbers is 38. If two numbers, namely 45 and 55 are discarded, the average of the remaining numbers is (CO5) 6

SECTION C **50**

4. Answer any <u>one</u> of the following:-

4-a. Determine the value of λ and μ so that the equations $\begin{aligned} x+y+z &= 6 \\ x+2y+3z &= 10 \\ x+2y+\lambda z &= \mu \end{aligned}$ have (i) no solution (ii) a unique solution (iii) infinite many solutions. (CO1) 10

4-b. Verify Caley-Hamilton Theorem for the matrix $\begin{bmatrix} 3 & 1 & -1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. (CO1) 10

5. Answer any <u>one</u> of the following:-

5-a. If $y = \sin(a \sin^{-1} x)$, then show that $(1-x^2)\, y_2 - x y_1 + a^2 y = 0$ and $(1-x^2)\, y_{n+2} - (2n+1)\, x y_{n+1} - (n^2 - a^2) y_n = 0$, (CO2) 10

5-b. Trace the curve $r^2 = a^2 \cos 2\theta$ (CO2) 10

6. Answer any <u>one</u> of the following:-

6-a. Expand the function $\tan^{-1}\left(\frac{y}{x}\right)$ in the neighbourhood of (1, 1) upto and inclusive of second degree terms. Hence compute the value of the function at (1.1, 0.9) approximately. (CO3) 10

6-b. Find the maximum and minimum distances from the origin to the curve $x^2 + 4xy + 6y^2 = 140$. (CO 3) 10

7. Answer any <u>one</u> of the following:-

7-a. Evaluate $\iiint x^2 yz\, dx\, dy\, dz$, throughout the volume bounded by the planes 10

$x = 0, y = 0, z = 0 \text{ and } \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (CO 4)

7-b. Evaluate $\int\int_R y \, dxdy$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$. (CO 4) 10

8. Answer any <u>one</u> of the following:-

8-a. (i) If the radius of the cylinder increases by 10 % and the height increases by 20%. Then, what is the change in the volume of the cylinder ? 10

(ii) The average age of eight teachers in a school is 40 years. A teacher among them died at the age of 55 years whereas another teacher whose age was 39 years joins them. The average age of the teachers in the school now is (in years)

(iii) A machine is sold for Rs5060 at a gain of 10%. What would have been the gain or loss % if it had been sold for Rs 4370?

(CO5)

8-b. (i) If the price of an item is decreased by 10% and then increased by 10%, the net effect on the price of the item is: 10

(ii) The average marks obtained by 40 students of a class is 86. If the 5 highest marks are removed, the average reduced by one marks. The average marks of the top 5 students is ?

(iii) Find the missing terms: 1, 2, 6, 7, 21, 22, 66, 67, ?

(CO5)