## Roll. No:

# NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA 

(An Autonomous Institute Affiliated to AKTU, Lucknow)
M.Tech (Integrated)

SEM: IV - THEORY EXAMINATION (20.....-20.....)
Subject: Engineering Mathematics- IV
Time: 3 Hours
Max. Marks: 100

## General Instructions:

IMP: Verify that you have received the question paper with the correct course, code, branch etc.

1. This Question paper comprises of three Sections -A, B, \& C. It consists of Multiple Choice Questions (MCQ's) \& Subjective type questions.
2. Maximum marks for each question are indicated on right -hand side of each question.
3. Illustrate your answers with neat sketches wherever necessary.
4. Assume suitable data if necessary.
5. Preferably, write the answers in sequential order.
6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

## SECTION A

## 1. Attempt all parts:-

1-a. Curve which are more sharply peaked than normal curve is called: (CO1)
(a) Leptokurtic curve
(b) Mesokurtic curve
(c) Platykurtic curve
(d) None of these

1-b. Two lines of regression are $x+2 y-5=0,2 x+3 y-8=0$ then mean value of $x$ and $y$, 1 are respectively-(CO1)
(a) 4,7
(b) 1,2
(c) $-1,-2$
(d) None of these

1-c. Any hypothesis which is tested for the purpose of rejection under the 1 assumption that it is true, is called (CO2)
(a) Null Hypothesis
(b) Alternate hypothesis
(c) Statistical hypothesis
(d) Composite hypothesis

1-d. In ANOVA, when calculated value of F is greater than the tabulated value, then the null hypothesis is? (CO2)
(a) Accepted
(b) Rejected
(c) Data insufficient.
(d) None of these.

1-e. Let $X$ and $Y$ be the jointly continuous random variables with joint CDF satisfies the following condition (CO3)
(a) $\operatorname{Fxy}(-\infty, \infty)=1$
(b) $F x y(x,-\infty)=1$
(c) $\operatorname{Fxy}(-\infty, y)=1$
(d) None of these

1-f. If $X$ and $Y$ are two random variables having joint density function:
$f(x, y)= \begin{cases}\frac{1}{8}(6-x-y) & 0 \leq x<2, \quad 2 \leq y<4 \\ 0 & \text { otherwise }\end{cases}$
then $P(X<1 \mid Y<3)$ :
(a) $6 / 5$
(b) $3 / 5$
(c) $2 / 5$
(d) $1 / 5$

1-g. The moment generating function about origin of binomial distribution is $\left(q+p e^{t}\right)^{n}$, Then the mean and variance of Binomial distribution :(CO4)
(a) np, npq
(b) npq, np
(c) $\mathrm{np}, \mathrm{np}$
(d) npq, npq

1-h. If $X$ has a Poisson Distribution with $P(X=0)=P(X=1)$, then value of $E(X)$ is: (CO4)
(a) 0
(b) 1
(c) 2
(d) None of these

1-i. Function $f(x)=x^{3} \sin x$ is: (CO5)
(a) Odd
(b) Even
(c) Neither odd nor even
(d) None of these

1-j. The sum of squares of three numbers is 138 and the sum of their products taken two at a time is 131 . Find their sum. (CO5)
(a) 35
(b) 42
(c) 20
(d) 18

## 2. Attempt all parts:-

2.a. Write down the angle between two lines of regression. (CO1)
2.b. What is test statistic for difference of mean for large sample? (CO2)
2.c. Calculate variance of $x$. if the probability distribution of the random variable $x$ is given by (CO3)

| $x$ | -1 | 0 | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $f$ | 0.3 | 0.1 | 0.1 | 0.3 | 0.2 |

2.d. Consider a random variable with exponential distribution with $\lambda=1$ Compute the probability for $\mathrm{P}(\mathrm{X}>3$ ). (CO4)
2.e. How many words can be formed by using all letters of the word "MATHEMATICS"? (CO5)

## SECTION B

## 3. Answer any five of the following:-

3-a. Calculate the correlation coefficient between X and Y from the following data-(CO1)

| X | 3 | 7 | 5 | 4 | 6 | 8 | 2 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 7 | 12 | 8 | 8 | 10 | 13 | 5 | 10 |

3-b. The following results were obtained from the record of age $(x)$ and the blood pressure(y) of a group of 10 men:

|  | $x$ | $y$ |
| :--- | :--- | :--- |
| Mean | 53 | 142 |
| Variance | 130 | 165 |

and $\sum(\mathrm{x}-\overline{\mathrm{x}})(\mathrm{y}-\overline{\mathrm{y}})=1220$. Find the appropriate regression equation and use it to estimate the blood pressure of a man whose age is 45 .(CO1)

3-c. The following table gives the number of accidents that took place in an industry during various days of a week. Test if accidents are uniformly distributed over the week (CO2)

| Day | Mon | Tue | Wed | Thu | Fri | Sat |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| No. of <br> accidents | 14 | 18 | 12 | 11 | 15 | 14 |

Given that tabular value of Chi-Square at $5 \%$ LOS for 5 degree of freedom is 11.09.

3-d. In a test given to two groups of students, the marks obtained are as follows: (CO2)
First Group 182036504936344941
Second Group 29282635304446 - -
Examine the significance of difference between the mean marks secured by students of the above two groups. (Given as $5 \%$ level of significance, the value for 14 d.f. $=2.14$ ).
3.e. The diameter, say $X$, of a electric cable, is assumed to be continuous random variable with pdf $f(x)=6 x(1-x), 0 \leq x \leq 1$ (CO3)
i) Obtain an expression for the c.d.f. of X .
ii) Determine the number k such that $\mathrm{P}(\mathrm{X}<\mathrm{k})=\mathrm{P}(\mathrm{X}>\mathrm{k})$.
3.f. Net profit of 400 companies is normally distributed with a mean profit of Rs. 150 lakhs and a standard deviation of Rs. 20 lakhs. Find the number of companies whose profits(Rs. Lakhs ) are between 100 and 138.Also find the minimum profit of top $15 \%$ companies. (Area for $\mathrm{Z}=2.5,1.04$ and 0.6 are $0.4938,0.35$ and 0.2251). (CO4)
3.g. Find the unit digit of (4137) ${ }^{754}$ (CO5)

## SECTION C

## 4. Answer any one of the following:-

4-a. The following table represent the height of a batch of 100 students. Calculate 10 kurtosis: (CO1)

| Height <br> (in cm) | 59 | 61 | 63 | 65 | 67 | 69 | 71 | 73 | 75 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| No. of <br> studen <br> ts | 0 | 2 | 6 | 20 | 40 | 20 | 8 | 2 | 2 |

4-b. Fit a relation $y=a+\frac{b}{x}+\frac{c}{x^{2}}$ for the following data: (CO1)

| x | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 15.3 | 15.1 | 15 | 14.5 | 14 |

## 5. Answer any one of the following:-

5-a. (i) A sample of heights of 6400 English men has a mean of 170 cm and SD of 6.4 cm, while a sample of heights of 1600 Americans has a mean of 172 cm and SD of 6.3 cm . Do the data indicate that Americans are, on the average, taller than the Englishmen? Given that the tabular value for test is 2.33 at $1 \%$ LOS.
(ii)Write a short note on SQC.(CO2)

5-b. A manufacturing company purchased three new machines of different makes and wishes to determine whether one of them is faster than the others in producing a certain output. Five hourly production figures are observed at random from each machines and results are given below: (CO2)

| Observation | $\mathrm{A}_{1}$ | $\mathrm{~A}_{2}$ | $\mathrm{~A}_{3}$ |
| :--- | :--- | :--- | :--- |
| 1 | 25 | 31 | 24 |
| 2 | 30 | 39 | 30 |
| 3 | 36 | 38 | 28 |
| 4 | 38 | 42 | 25 |
| 5 | 31 | 35 | 28 |

Use ANOVA and determine whether the machines are significantly different in their mean speed. (Given : at $5 \%$ level , $\mathrm{F}(2,12)=3.89$ )

## 6. Answer any one of the following:-

6-a. The joint pdf of a two dimensional Random variable $(X, Y)$ is: (CO3)
$f(x, y)=x y^{2}+\frac{x^{2}}{8}, 0 \leq x \leq 2,0 \leq y \leq 1$
i) find marginal distribution of $X$ and $Y$
ii) find conditional distribution of $X$ given $Y$ and $Y$ given $X$.

6-b. A random variable $X$ has the following probability function: (CO3)

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $p(x)$ | 0 | $k$ | $2 k$ | $2 k$ | $3 k$ | $k^{2}$ | $2 k^{2}$ | $7 k^{2}+k$ |

Then (i) find $k$
(ii) Evaluate $P(X<6), P(X \geq 6), P(3<X \leq 6)$
(iii) Find the minimum value of $x$ so that $P(X \leq x)>1 / 2$

## 7. Answer any one of the following:-

7-a. In 800 families with 5 children each, how many families would be expected to have- (CO4)
I. 3 boys and 2 girls
II. 2 boys and 3 girls
III. No girl
IV. At most 2 girls. (Assume probabilities for boys and girls to be equal)

7-b. Fit a Poisson distribution to the set of observations:(CO4)

| $x$ | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $f(x)$ | 122 | 60 | 15 | 2 | 1 |

## 8. Answer any one of the following:-

8-a. The probability that machine A will be performing an usual function in 5 years time is $1 / 4$ while the probability that machine $B$ will be operating usually at the end of the same period is $1 / 3$. Find the probability in the following cases that in 5 years time: (CO5)
i. Both machines will be performing an usual function
ii. Neither will be operating
iii. Only machine B will be operating
iv. At least one of the machines will be operating

8-b. (A) .If the number 3422213 pq is divisible by 99, find the missing digits $p$ and $q$. 10 (CO5)
(B) Find the remainder when (397) $3589+5$ is divided by 398

