Printed Page:- **Subject Code:- AAS0401B**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: IV - THEORY EXAMINATION (2022-2023)

Subject: Engineering Mathematics- III

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* **three Sections -A, B, & C.** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION A **20**

1. Attempt all parts:-

1-a. $\lim_{z \to 0} \frac{\bar{z}}{z}$ is: (CO1) 1

(a) Depend on path.

(b) Independent of path.

(c) does not exist.

(d) None of these.

1-b. The function $f(z) = e^x(\cos y + i \sin y)$ is : (CO1) 1

(a) Analytic everywhere.

(b) Nowhere Analytic.

(c) Analytic at origin only.

(d) None of these.

1-c. $\int_0^1 z\bar{z}\, dz$ along the path $y^2 = x$ is equal to: (CO2) 1

(a) $(\frac{6}{5} + \frac{15}{8}i)$

(b) $(\frac{5}{6} - \frac{8}{15}i)$

(c) $(\frac{6}{5} - \frac{8}{15}i)$

(d) $(\frac{5}{6} + \frac{8}{15}i)$

1-d. If C is closed curve $|z| = r$ and n ≠ -1 then the value of $\int_C z^n \, dz$ is : (CO2) 1

(a) $2\pi i$

(b) 0

(c) $4\pi i$

(d) $2i$

1-e. What is the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ when the ratio is negative? (CO3) 1

(a) $u = (c_1 \, e^{px} + c_2 e^{-px}) c_3 e^{p^2 c^2 t}$

(b) $u = (c_1 \, x + c_2) c_3$

(c) $u = (c_1 \, Cospx + c_2 Sinpx) c_3 e^{-p^2 c^2 t}$

(d) None of these

1-f. Solution of PDE $z_{xx} - z_{xy} - 6z_{yy}=0$ is: (CO3) 1

(a) $Z=f_1(y-2x) + f_2(y+3x)$

(b) $Z=f_1(y+2x) + f_2(y-3x)$

(c) $Z=f_1(y-2x) + f_2(y-3x)$

(d) None of these

1-g. The Fourier Transform of $F(x) = e^{-a|x|}, a > 0.$ (CO4) 1

(a) $\frac{2a}{p^2 - a^2}$

(b) $\frac{2a}{p^2 + a^2}$

(c) $\frac{2a}{a^2 - p^2}$

(d) $\frac{a}{a^2 + p^2}$

1-h. $Z-$Transform of sequence $\{ab^k\}$ is: (CO4) 1

(a) $a\frac{z}{z-b}$

(b) $b\frac{z}{z-a}$

(c) $a\frac{z}{z-a}$

(d) $\frac{z}{z-b}$

1-i. Guddi's swimming speed in still water to the speed of river is 7:1. She swims 4.2 km up the river in just 14 min. How much time will Guddi take to swim 18.4 km down the river? (CO5) 1

(a) 46 minutes

(b) 12 minutes

(c) 23 minutes

(d) None of these

1-j. What will be angle between the two hands of a clock at 10:25 pm? (CO5) 1

(a) 120°

(b) 146.5°

(c) 162.5°

(d) None of these

2. Attempt all parts:-

2.a. Check that the $\lim_{z\to 0}\left(\frac{z}{\bar{z}}\right)^2$ is exists or not ? (CO1) 2

2.b. Evaluate: $\oint_c \frac{z^2-z+1}{z-2}dz$; where $C \equiv |z-1| = \frac{1}{2}$. (CO2) 2

2.c. Classify the PDE: $yu_{xx}+(x+y)u_{xy}+xu_{yy}=0$ about the line $y=x$. (CO3) 2

2.d. Find z-transform of unit step function. (CO4) 2

2.e. At what time between 2 and 3 o'clock will the hands of a clock be together? (CO5) 2

SECTION B **30**

3. Answer any <u>five</u> of the following:-

3-a. Determine an analytic function $f(z)$ in terms of z whose real part is $e^{-x}(x\cos y+y\sin y)$; $f(0)=1$. (CO1) 6

3-b. Find the image of infinite strip $\frac{1}{4}\le y\le\frac{1}{2}$ under the transformation $w=\frac{1}{z}$ .Also show the region graphically. (CO1) 6

3-c. Evaluate $\oint_C (y-x-3x^2i)dz$; where C is a straight line from $z=0$ to $z=1+i$. (CO2) 6

3-d. Expand $f(z)=\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in Laurent series valid for the region $3<|z+2|<5$. (CO2) 6

3.e. Solve the PDE: $(D^2-DD')z=\sin x.\cos y$. (CO3) 6

3.f. Find the Fourier transform of $F(x)=e^{-ax^2}$, $a>0$. (CO4) 6

3.g. Time taken by A to finish a piece of work is twice the time taken B and thrice 6

the time taken by C. If all three of them work together, it takes them 2 days to complete the entire work. How much work was done by B alone? (CO5)

SECTION C **50**

4. Answer any <u>one</u> of the following:-

4-a. If f(z) is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|\text{Ref}(z)|^2 = 2|f'(z)|^2$. (CO1) 10

4-b. Find the transformation which maps the points $z = 1, -i, -1$ into the points $w = i, 0, -i$ respectively. 10

Also show that the transformation maps the region outside the circle $|z| = 1$ in to the half space $R(w) \geq 0$. (CO1)

5. Answer any <u>one</u> of the following:-

5-a. Verify the Cauchy integral theorem for the function $f(z) = z^2$ taken over the boundaries of a Square with vertices $1 \pm i$, $-1 \pm i$. (CO2) 10

5-b. Evaluate $\int_0^{\pi} \frac{d\theta}{3+\sin^2\theta}$ using contour integration. (CO2) 10

6. Answer any <u>one</u> of the following:-

6-a. Solve the PDE: $(D^2 - 3DD' + 2D'^2)z = e^{2x-y} + e^{x+y} + \cos(x+2y)$. (CO3) 10

6-b. Solve the PDE: $(3D^2 - 2D'^2 + D - 1)z = 4e^{x+y}\cos(x+y)$. (CO3) 10

7. Answer any <u>one</u> of the following:-

7-a. Solve by Z − transform: $y_{k+3} - 3y_{k+2} + 3y_{k+1} - y_k = u(k);\ y(0) = y(1) = y(2) = 0$. (CO4) 10

7-b. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $x > 0, t > 0$ by using suitable fourier transform, subject to the conditions 10

(i) $u = 0$ when $x = 0, t > 0$

(ii) $u = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$ when $t = 0$

(iii) $u(x, t)$ is bounded. (CO4)

8. Answer any <u>one</u> of the following:-

8-a. **(i)** Bucket P has thrice the capacity as bucket Q. It takes 60 turns for bucket P to fill the empty drum. How many turns will it take for both the buckets P and Q, having each turn together to fill the empty drum? 10

(ii) Two pipes A and B can fill a tank in 15 hours and 20 hours respectively while a third pipe C can empty the full tank in 25 hours. All the three pipes are opened in the beginning. After 10 hours, C is closed. In how much time will the tank be full? (CO5)

8-b. **(i)** Two pipes A and B together can fill a cistern in 4 hours. Had they been opened separately, then B would have taken 6 hours more than A to fill the 10

cistern. How much time will be taken by A alone to fill the cistern?

(ii) 16 men can finish a work in 24 days and 48 boys can finish the same work in 16 days. 12 men started the work and after 4 days 12 boys joined them. In how many days can they finish the remaining work? (CO5)