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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M. Tech (Integrated)

(SEM: III-THEORY EXAMINATION (2022-2023))

Subject: Engineering Mathematics-III

Time: 3Hours

Max. Marks:100

General Instructions:

IMP: Verify that you have received question paper with correct course, code, branch etc.

1. This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.
2. Maximum marks for each question are indicated on right hand side of each question.
3. Illustrate your answers with neat sketches wherever necessary.
4. Assume suitable data if necessary.
5. Preferably, write the answers in sequential order.
6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION – A

20

1. Attempt all parts:-

- 1-a. Find the value of c_1 and c_2 such that the function $f(z) = x^2 + c_1y^2 - 2xy + i(c_2x^2 - y^2 + 2xy)$ is analytic. (CO1) 1
- (a) 1, 1
(b) 1, -1
(c) -1, 1
(d) None of these
- 1-b. A function is said to be harmonic if (CO1) 1
- (a). $u_{xx} + u_{yy} = 0$
(b). $u_{xy} + u_{yy} = 0$
(c). $u_{xx} - u_{yy} = 0$
(d). None of these
- 1-c. If there is no pole inside and on the contour, then the value of integral is (CO2) 1
- (a). ∞
(b). $2\pi i$
(c). 0

(d).None of these

- 1-d. For the function $\frac{\sin z}{z^3}$ of a complex variable z , the point $z = 0$ is a (CO2) 1
- (a). Pole of order 2
(b). Pole of order 3
(c). Pole of order 1
(d).None of these
- 1-e. The Particular integral of PDE $(D^2 - DD' + D' - 1)z = \cos(x + 2y)$ is (CO3) 1
- (a) $z = \frac{1}{2}\cos(x + 2y)$
(b) $z = \frac{1}{2}\sin(x + 2y)$
(c) $z = \frac{x}{2}\cos(x + 2y)$
(d)None of these
- 1-f. The solution of Differential equation $(D + 1)(D + D' - 1) = 0$ is (CO3) 1
- (a). $e^{-x}\phi(y) + e^x\phi_2(y - x)$
(b). $e^{-x}\phi(y) + e^x\phi_2(y + x)$
(c). $e^x\phi(y) + e^x\phi_2(y - x)$
(d).None of these
- 1-g. The order of convergence of Newton Raphson method is (CO4) 1
- (a). 0
(b). 2
(c). 3
(d).None of these
- 1-h. In Crout's method elements of upper triangular matrix is taken as (CO4) 1
- (a) 1, 1, 1
(b) 0, 0, 0
(c) 1, 0, 1
(d) None of these
- 1-i. The average speed of a car which covers half the distance with a speed of 20 km/hr and the other half with the speed of 30 km/hr will be (CO5) 1
- (a). 50 km/hr
(b). 24 km/hr
(c). 26 km/hr
(d).None of these
- 1-j. A can finish a work in 18 days and B can do the same work in half the time taken by A. Then, working together, what part of the same work they can finish in a day? (CO5) 1
- (a) $1/6$
(b) $1/9$
(c) $2/5$
(d) None of these

2. Attempt all parts:-

- 2.a. Find analytic function whose real part is $e^x \cos y$. ? (CO1) 2
- 2.b. Discuss the singularity of $f(z) = \sin \frac{1}{1-z}$ at $z = 1$. (CO2) 2
- 2.c. Classify the following equation: $2 \frac{\partial^2 u}{\partial x^2} + 6 \frac{\partial^2 u}{\partial x \partial y} + 3 \frac{\partial^2 u}{\partial y^2} = 0$? (CO3) 2
- 2.d. Evaluate $\Delta^2 \sin x$ taking $h=1$, symbols have their usual meanings.? (CO4) 2
- 2.e. Pipes A and B can fill a tank in 5 and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the tank will be filled in..... hrs.? (CO5) 2

SECTION – B

30

3. Answer any five of the following-

- 3-a. Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic. Express $f(z) = u + iv$ as an analytic function of z . (CO1) 6
- 3-b. If $f(z)$ is a regular function of z , prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$ (CO1) 6
- 3-c. Use Cauchy Integral Formula to evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C is $|z| = 3$. (CO2) 6
- 3-d. Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along the paths. (CO2) 6
- i. $y = x$
ii. $y = x^2$
- 3-e. Solve the PDE: $(D^2 - DD')z = \sin(x + 2y)$ (CO3) 6
- 3-f. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's 1/3rd rule. Hence obtain the approximate value of π dividing the range into 6 equal parts.. (CO4) 6
- 3-g. The speed of the boat in still water is 5 times that of the current, it takes 1.1 hours to row to point B from point A downstream. The distance between point A and point B is 13.2km. How much distance (in km) will it cover in 312 minutes upstream? (CO5) 6

SECTION – C

50

4. Answer any one of the following-

- 4-a. Examine the nature of the function 10
- $$f(z) = \begin{cases} \frac{x^3 y (y - ix)}{x^6 + y^2}, & z \neq 0 \\ 0 & z = 0 \end{cases}, \text{ prove that } \frac{f(z) - f(0)}{z} \rightarrow 0 \text{ as } z \rightarrow 0 \text{ along}$$
- any radius vector but not as $z \rightarrow 0$ in any manner and also that $f(z)$ is not analytic at $z=0$. (CO1)

- 4-b. Find the transformation which maps the points $z = 1, -i, -1$ to the points $w = i, 0, -i$ respectively. Show also that this transformation maps the region outside the circle $|z| = 1$ into the half plane $\operatorname{Re}(w) \geq 0$. (CO1) 10

5. Answer any one of the following-

- 5-a. State Laurent's theorem. Find the series expansion of $f(z) = \frac{z^2-1}{z^2+5z+6}$ about $z = 0$ in the region (i) $|z| < 2$ (ii) $2 < |z| < 3$. (CO2) 10

- 5-b. State Cauchy's Residue theorem and hence evaluate (CO2) 10

$$\int_0^{2\pi} \frac{\cos 2\theta}{1-2a \cos \theta + a^2} d\theta, \quad a^2 < 1.$$

6. Answer any one of the following-

- 6-a. Solve: $D(D - 2D' - 3)z = e^{x+2y}$, where $D = \frac{\partial}{\partial x}$ & $D' = \frac{\partial}{\partial y}$. (CO3) 10

- 6-b. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3 \frac{\pi x}{l}$. If it is released from rest from this position, find the displacement $y(x, t)$. (CO3) 10

7. Answer any one of the following-

- 7-a. Evaluate $y(0.2)$ if $\frac{dy}{dx} = -2xy^2$ and $y(0)=1$ using Runge-Kutta method of second order by taking $h=0.2$. Also compare the result with exact value. (CO4) 10

- 7-b. Find $f(x)$ as polynomial in x for the following data: (CO4) 10

X	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335

8. Answer any one of the following-

- 8-a. How many number of times 29th day of the month occurs in 400 consecutive year. (CO5) 10

- 8-b. A builder appoints three construction workers Akash, Sunil and Rakesh on one of his sites. They take 20, 30 and 60 days respectively to do a piece of work. How many days will it take Akash to complete the entire work if he is assisted by Sunil and Rakesh every third day? (CO5) 10