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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

(SEM: III-THEORY EXAMINATION (2022-2023)

Subject: Engineering Mathematics-III

Time: 3Hours   Max. Marks:100

General Instructions:

IMP: Verify that you have received question paper with correct course, code, branch etc.

1. *This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*
2. *Maximum marks for each question are indicated on right hand side of each question.*
3. *Illustrate your answers with neat sketches wherever necessary.*
4. *Assume suitable data if necessary.*
5. *Preferably, write the answers in sequential order.*
6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION – A   20

1. Attempt all parts:-

1-a.   Find the value of c_1 and c_2 such that the function $f(z) = x^2 + c_1 y^2 - 2xy + i(c_2 x^2 - y^2 + 2xy)$ is analytic. (CO1)   1

        (a) $1, 1$
        (b) $1, -1$
        (c) $-1, 1$
        (d) None of these

1-b.   A function is said to be harmonic if (CO1)   1

        (a). $u_{xx} + u_{yy} = 0$

        (b). $u_{xy} + u_{yy} = 0$

        (c). $u_{xx} - u_{yy} = 0$

        (d). None of these

1-c.   If there is no pole inside and on the contour, then the value of integral is (CO2)   1
        (a). ∞

        (b). $2\pi i$

        (c). 0

(d). None of these

1-d. For the function $\frac{\sin z}{z^3}$ of a complex variable z, the point $z = 0$ is a (CO2)   1

   (a). Pole of order 2

   (b). Pole of order 3

   (c). Pole of order 1

   (d). None of these

1-e. The Particular integral of PDE $(D^2 - DD' + D' - 1)z = \cos(x + 2y)$ is (CO3)   1

   (a) $z = \frac{1}{2}\cos(x + 2y)$

   (b) $z = \frac{1}{2}\sin(x + 2y)$

   (c) $z = \frac{x}{2}\cos(x + 2y)$

   (d) None of these

1-f. The solution of Differential equation $(D + 1)(D + D' - 1) = 0$ is (CO3)   1

   (a). $e^{-x}\emptyset(y) + e^x\emptyset_2(y - x)$

   (b). $e^{-x}\emptyset(y) + e^x\emptyset_2(y + x)$

   (c). $e^x\emptyset(y) + e^x\emptyset_2(y - x)$

   (d). None of these

1-g. The order of convergence of Newton Raphson method is (CO4)   1

   (a). 0

   (b). 2

   (c). 3

   (d). None of these

1-h. In Crout's method elements of upper triangular matrix is taken as (CO4)   1
   (a) 1, 1, 1
   (b) 0, 0, 0
   (c) 1, 0, 1
   (d) None of these

1-i. The average speed of a car which covers half the distance with a speed of 20   1

   km/hr and the other half with the speed of 30 km/hr will be (CO5)

   (a). 50 km/hr

   (b). 24 km/hr

   (c). 26 km/hr

   (d). None of these

1-j. A can finish a work in 18 days and B can do the same work in half the time taken by A.   1
   Then, working together, what part of the same work they can finish in a day? (CO5)
   (a) 1/6
   (b) 1/9
   (c) 2/5
   (d) None of these

2. Attempt all parts:-

2.a. Find analytic function whose real part is $e^x \cos y$. ? (CO1)   2

2.b. Discuss the singularity of $f(z) = \sin \frac{1}{1-z}$ at $z = 1$. (CO2)   2

2.c. Classify the following equation: $2\frac{\partial^2 u}{\partial x^2} + 6\frac{\partial^2 u}{\partial x \partial y} + 3\frac{\partial^2 u}{\partial y^2} = 0$? (CO3)   2

2.d. Evaluate $\Delta^2 \sin x$ taking h=1, symbols have their usual meanings.? (CO4)   2

2.e. Pipes A and B can fill a tank in 5 and 6 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, then the tank will be filled in…… hrs.? (CO5)   2

<div align="center">SECTION – B   30</div>

3. Answer any <u>five</u> of the following-

3-a. Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic. Express $f(z) = u+iv$ as an analytic function of z .(CO1)   6

3-b. If $f(z)$ is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ (CO1)   6

3-c. Use Cauchy Integral Formula to evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C is $|z| = 3$. (CO2)   6

3-d. Evaluate $\int_0^{1+i}(x^2 - iy)dz$ along the paths.(CO2)   6

      i. $y = x$
     ii. $y = x^2$

3-e. Solve the PDE: $(D^2 - DD')z = \sin(x + 2y)$ (CO3)   6

3-f. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's 1/3rd rule. Hence obtain the approximate value of π dividing the range into 6 equal parts..(CO4)   6

3-g. The speed of the boat in still water is 5 times that of the current, it takes 1.1 hours to row to point B from point A downstream. The distance between point A and point B is 13.2km. How much distance (in km) will it cover in 312 minutes upstream? (CO5)   6

<div align="center">SECTION – C   50</div>

4. Answer any <u>one</u> of the following-

4-a. Examine the nature of the function   10

$$f(z) = \begin{cases} \dfrac{x^3 y(y-ix)}{x^6 + y^2}, & z \neq 0 \\ 0 & z = 0 \end{cases}, \text{ prove that } \dfrac{f(z) - f(0)}{z} \to 0 \text{ as } z \to 0 \text{ along}$$

any radius vector but not as $z \to 0$ in any manner and also that $f(z)$ is not analytic at $z = 0$.(CO1)

4-b.   Find the transformation which maps the points $z = 1, -i, -1$ to the points $w = i, 0, -i$ 10
respectively. Show also that this transformation maps the region outside the circle $|z| = 1$ into the half plane $Re(w) \geq 0$.(CO1)

5. Answer any <u>one</u> of the following-

5-a.   State Laurent's theorem. Find the series expansion of $f(z) = \dfrac{z^2-1}{z^2+5z+6}$ about $z = 0$ in   10
the region (i) $|z| < 2$ (ii) $2 < |z| < 3$.(CO2)

5-b.   State Cauchy's Residue theorem and hence evaluate (CO2)   10
$\int_0^{2\pi} \dfrac{Cos\ 2\theta}{1-2a\ cos\theta+a^2}\ d\theta,\ a^2 < 1.$

6. Answer any <u>one</u> of the following-

6-a.   Solve:$D(D - 2D' - 3)z = e^{x+2y}$, where $D = \dfrac{\partial}{\partial x}$ & $D' = \dfrac{\partial}{\partial y}$.(CO3)   10

6-b.   A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a   10
position given by $y = y_0\ sin^3\dfrac{\pi x}{l}$. If it is released from rest from this position,
find the displacement $y(x, t)$. (CO3)

7. Answer any <u>one</u> of the following-

7-a.   Evaluate y(0.2) if $\dfrac{dy}{dx} = -2xy^2$ and y(0)=1 using Runge-Kutta method of second   10
order by taking h=0.2. Also compare the result with exact value. (CO4)

7-b.   Find f(x) as polynomial in x for the following data: (CO4)   10

X	-4	-1	0	2	5
f(x)	1245	33	5	9	1335

8. Answer any <u>one</u> of the following-

8-a.   How many number of times 29th day of the month occurs in 400   10
consecutive year.(CO5)

8-b.   A builder appoints three construction workers Akash, Sunil and Rakesh on one   10
of his sites. They take 20, 30 and 60 days respectively to do a piece of work.
How many days will it take Akash to complete the entire work if he is assisted
by Sunil and Rakesh every third day? (CO5)