Printed Page:- Subject Code:- AMICSE0306
Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA
(An Autonomous Institute Affiliated to AKTU, Lucknow)
M. Tech.
SEM: III - THEORY EXAMINATION (2021 - 2022)
Subject: Discrete Structures

Time: 03:00 Hours Max. Marks: 100

General Instructions:

1. All questions are compulsory. It comprises of three Sections A, B and C.

- Section A - Question No- 1 is objective type question carrying 1 mark each & Question No- 2 is very short type questions carrying 2 marks each.
- Section B - Question No- 3 is Long answer type - I questions carrying 6 marks each.
- Section C - Question No- 4 to 8 are Long answer type - II questions carrying 10 marks each.
- No sheet should be left blank. Any written material after a Blank sheet will not be evaluated/checked.

SECTION A 20

1. Attempt all parts:-

1-a. The binary relation {(1,1), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2)} on the set {1, 2, 3} is __________. (CO1) 1

1. reflective, symmetric and transitive
2. irreflexive, symmetric and transitive
3. neither reflective, nor irreflexive but transitive
4. irreflexive and antisymmetric

1-b. Which of the following function f : Z X Z → Z is not onto? (CO1) 1

1. f(a, b) = a + b
2. f(a, b) = a
3. f(a, b) = |b|
4. f(a, b) = a – b

1-c. An identity element of a group has ______ element. (CO2) 1

1. associative
2. commutative
3. inverse
4. homomorphic

1-d. If x * y = x + y + xy then (G, *) is _____________ . (CO2) 1

1. Monoid
2. Abelian group
3. Commutative semigroup
4. Cyclic group

1-e. Let D30 = {1, 2, 3, 4, 5, 6, 10, 15, 30} and relation be partial ordering on D30. The all lower bounds of 10 and 15 respectively are (CO3) 1

1. 1,3
2. 1,5
3. 1,3,5

4. None of these

1-f. A Poset in which every pair of elements has both a least upper bound and a greatest lower bound is termed as _______ . (CO3) 1

1. lattice
2. sublattice
3. trail
4. walk

1-g. Let P: If Sahil bowls, Saurabh hits a century.; Q: If Raju bowls, Sahil gets out on first ball. Now if P is true and Q is false then which of the following can be true? (CO4) 1

1. Raju bowled and Sahil got out on first ball
2. Raju did not bowled
3. Sahil bowled and Saurabh hits a century
4. Sahil bowled and Saurabh got out

1-h. $A \rightarrow (A \vee q)$ is a _________ . (CO4) 1

1. Tautology
2. Contradiction
3. Contingency
4. None of the mentioned

1-i. Let G be the non-planar graph with minimum possible number of edges. Then G has (CO5) 1

1. 9 edges and 5 vertices
2. 9 edges and 6 vertices
3. 10 edges and 5 vertices
4. 10 edges and 6 vertices

1-j. The balance factor of a node in a binary tree is defined as (CO5) 1

1. addition of heights of left and right subtree
2. height of right subtree minus height of left subtree.
3. height of left subtree minus height of right subtree
4. height of right subtree minus one

2. Attempt all parts:-

2-a. Represent $(A \oplus B)$ with venn diagram. (CO1) 2

2-b. If (G, *) is a group and a is an element in G, such that a * a = a , then show that a = e , where e is identity element in G. (CO2) 2

2-c. Find the glb and lub of the sets {3,9,12} and {1,2,4,5,10} if they exist in the poset $(Z^+, |)$. (C03) 2

2-d. Show that the propositions $P \rightarrow Q$ and $\neg P \vee Q$ are logically equivalent. (CO4) 2

2-e. Define Euler graph. Give an example of Eulerian graph which is not Hamiltonian graph. (CO5) 2

SECTION B 30

3. Answer any <u>five</u> of the following:-

3-a. Let f be the function from {a, b, c} to {1, 2, 3} such that f(a) = 2, f(b) = 3, and f(c) = 1. Is f invertible, and if it is, what is its inverse? (CO1) 6

3-b. Find the solution of recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with $a_0=2$ and $a_1=7$. (CO1) 6

3-c. Let $G = (Z^2, +)$ be a group and let H be a subgroup of G where $H = \{(x, y) \mid x = y\}$. Find the left cosets of H in G. Here Z is the set of integers. (CO2) 6

3-d.	If M is set of all non singular matrices of order 'n x n', then show that M is a group w.r.t. matrix multiplication. Is (M, *) an abelian group? Justify your answer. (CO2)	6
3-e.	Find product of sum expansion of each of the following (CO3) (1). f(x,y,z) = (x+z)y (2). f(x,y,z) = x (3). f(x,y,z) = xy'	6
3-f.	What is a tautology, contradiction and contingency? Show that $(P \vee Q) \wedge (\neg P \vee R) \rightarrow (Q \vee R)$ is tautology, contradiction or contingency . (CO4)	6
3-g.	Prove that a connected graph G is Euler graph if and only if every vertex of G is of even degree. (CO5)	6

SECTION C 50

4. Answer any one of the following:-

4-a.	Prove that the binary relation {(1,1), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2)} on the set {1, 2, 3} is neither reflective, nor irreflexive but transitive. (CO1)	10
4-b.	Prove that $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\dots\dots\dots\dots+\frac{1}{\sqrt{n}} > \sqrt{n}$ for $n \geq 2$ using principle of mathematical induction. (CO1)	10

5. Answer any one of the following:-

5-a.	Consider the groups (G1, *) and (G2,⊗) with identity elements e1 and e2 respectively. If f : G1 → G2 is a group homomorphism, then prove that (CO2) i) f(e1) = e2 ii) $f(a^{-1}) = [f(a)]^{-1}$ iii) If H1 is a sub group of G1 and H2 = f(H1), then H2 is a sub group of G2 iv) If f is an isomorphism from G1 onto G2, then f^{-1} is an isomorphism from G2 onto G1.	10
5-b.	Let G be a group and let H be a subgroup of finite index. Then show that there exists a normal subgroup N of G such that N is of finite index in G and $N \subset H$. (CO2)	10

6. Answer any one of the following:-

6-a.	Let $(L, \vee, \wedge, \leq)$ be a distributive lattice and $a,b \in L$. if $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$ then show that b = c. (CO3)	10
6-b.	Given the Boolean expression X = AB + ABC + A B' C' + A C' (CO3) (i) Draw the logic diagram for the expression. (ii) Minimize the expression. (iii) Draw the logic diagram for the reduced expression.	10

7. Answer any one of the following:-

7-a.	Determine whether these are valid arguments. (CO4) i) If x is a positive real number, then x^2 is a positive real number. Therefore, if a^2 is positive, where a is a real number, then a is a positive real number. ii) If $x^2 = 0$, where x is a real number, then x = 0. Let a be a real number with $a^2 = 0$; then a = 0.	10
7-b.	Determine whether each of these compound propositions is satisfiable. (CO4) i) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$, ii) $(p \rightarrow q) \wedge (p \rightarrow \neg q) \wedge (\neg p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$, iii) $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$	10

8. Answer any one of the following:-

8-a.	Define planar graph. Prove that for any connected planar graph, v – e+ r = 2 Where v, e, r is the number of vertices, edges, and regions of the graph respectively. (CO5)	10
8-b.	Consider the graph G where V(G)= {A, B, C, D,E} and E(G)= {(A,D), (B,C) (C,E) (D,B) (D,D) (D,E)}. (CO5)	10

(i) Express G by its adjacency table.
(ii) Does G have any loop or parallel edges?
(iii) Find all simple path from D to E.
(iv) Find all cycles in G.
(v) Find the number of subgraphs of G with vertices C, D, E