

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute)

Affiliated to Dr. A.P.J. Abdul Kalam Technical University, Uttar Pradesh, Lucknow

B.Tech

SEM: I - THEORY EXAMINATION (2021 - 2022)

Subject: Mathematical Foundation - I

Time: 03:00 Hours

Max. Marks: 100

General Instructions:

1. All questions are compulsory. It comprises of three Sections A, B and C.
 - Section A - Question No- 1 is objective type question carrying 1 mark each & Question No- 2 is very short type questions carrying 2 marks each.
 - Section B - Question No- 3 is Long answer type - I questions carrying 6 marks each.
 - Section C - Question No- 4 to 8 are Long answer type - II questions carrying 10 marks each.
 - No sheet should be left blank. Any written material after a Blank sheet will not be evaluated/checked.

SECTION A

20

1. Attempt all parts:-

- 1-a. The eigen values of a matrix $\begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ are (CO1) 1
1. a, g, h
 2. a, h, c
 3. a, g, c
 4. a, b, c
- 1-b. By applying elementary transformation to a matrix, its rank is (CO1) 1
1. increases
 2. decreases
 3. does not change
 4. none of these
- 1-c. A subset W is called subspace of vector space $V(F)$ for $a, b \in F$ and $\alpha, \beta \in V$, if satisfy (CO2) 1
1. $a\alpha - b\beta \in V$
 2. $a\alpha \times b\beta \in V$
 3. $a\alpha \div b\beta \in V$
 4. $a\alpha + b\beta \in V$
- 1-d. If $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ is a linear transformation $T(1, 0) = (2, 3, 1)$ and $T(1, 1) = (3, 0, 2)$ then which of the following statement is correct ? (CO2) 1
1. $T(x y) = (x +y, 2x + y, 3x -3y)$
 2. $T(x y) = (2x +y, 3x -3y, x + y,)$
 3. $T(x y) = (2x -y, 3x +3y, x - y,)$
 4. $T(x y) = (x -y, 2x -y, 3x + 3y,)$
- 1-e. If the two tangents are real and coincide then origin is (CO3) 1
1. Node

2. cusp
3. Both node and cusp
4. None of these

1-f. If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{m}\right)^m$ then after two time differentiation, we get (CO3) 1

1. $x^2 y_2 + xy_1 + m^2 y = 0$
2. $(x^2 + 1)y_2 + xy_1 + m^2 y = 0$
3. $(x^2 + 1)y_2 - xy_1 - m^2 y = 0$
4. None of these

1-g. If $u = \sec^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$ then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \cot u$ (CO4) 1

1. 2
2. 1
3. 15
4. 18

1-h. Percentage error in the area of a rectangle when an error of +1 percent is made in measuring its length and breadth is given by (CO4) 1

1. 4%
2. 5%
3. 2%
4. 6%

1-i. A man had 7 children. When their average age was 12 years, a child aged 6 years died. The average age of remaining six children is (CO5) 1

1. 13 years
2. 10 years
3. 11 years
4. 14 years

1-j. Find the missing terms: 2, 11, 58, 295, 1482, ? (CO5) 1

1. 6750
2. 4450
3. 6459
4. 7419

2. Attempt all parts:-

2.a. Show that the matrix $\begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ is unitary matrix if $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$ (CO1). 2

2-b. The subset S { (1, 0, 0), (0, 1, 0), (0, 0, 1) } of the vector space R^3 is linearly..... (CO2) 2

2-c. Find y_n if, $y = \frac{x^n - 1}{x - 1}$ (CO3) 2

2-d. Verify the chain rule for Jacobians, if $x = u$, $y = u \tan v$ and $z = w$. (CO4) 2

2-e. A city has a population of 3,00,000, out of which 1,80,000 are males. 50% of the population is literate. If 70% of the males are literate, the number of literate females is ? (CO5) 2

SECTION B

30

3. Answer any five of the following:-

3-a. Find all values of μ for which rank of the matrix 6

$$\begin{bmatrix} \mu & -1 & 0 & 0 \\ 0 & \mu & -1 & 0 \\ 0 & 0 & \mu & -1 \\ -6 & 11 & -6 & 1 \end{bmatrix} \text{ is equal to 3. (CO1)}$$

- 3-b. Find the inverse of the matrix by using elementary transformation, where $\begin{bmatrix} i & -1 & 2i \\ 2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$. 6
(CO1)
- 3-c. Is the vector (3, -1, 0, -1) in the subspace of \mathbb{R}^4 spanned by the vectors (2, -1, 3, 2), (-1, 1, 1, -3) and (1, 1, 9, -5)? (CO2) 6
- 3-d. Consider the basis $S = \{a_1, a_2, a_3\}$ of \mathbb{R}^3 where $a_1 = (1, 1, 1)$, $a_2 = (1, 1, 0)$, $a_3 = (1, 0, 0)$. Express (2, -3, 5) in term of basis a_1, a_2, a_3 . (CO2) 6
- 3-e. Find the asymptotes of the curve $4x^3 - x^2y - 4xy^2 + y^3 + 3x^2 + 2xy - y^2 - 7 = 0$. (CO3) 6
- 3-f. Examine the function $f(x, y) = x^3 + y^3 - 3axy$ for maximum and minimum values. (CO4) 6
- 3-g. In certain code language 'si po re' means 'book is thick', 'ti na re' means 'bag is heavy', 'ka si' means 'interesting book' and 'de ti' means 'that bag'. What should stand for 'that is interesting' in that code language? (CO5) 6

SECTION C

50

4. Answer any one of the following:-

- 4-a. Find the characteristic equation of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence, compute A^{-1} . Also find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$. (CO1) 10
- 4-b. Show that the system of equations $3x + 4y + 5z = a$, $4x + 5y + 6z = b$, $5x + 6y + 7z = c$, does not have solution unless $a + c = 2b$. (CO1) 10

5. Answer any one of the following:-

- 5-a. Show that the mapping $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined as $T(a, b) = (a - b, b - a, -a)$ is a linear transformation. Find the range, null-space and nullity of T. (CO2) 10
- 5-b. In $V_2(\mathbb{F})$ define for $\alpha = (a_1, a_2)$ and $\beta = (b_1, b_2)$, $\langle \alpha, \beta \rangle = 2a_1\bar{b}_1 + a_1\bar{b}_2 + a_2\bar{b}_1 + a_2\bar{b}_2$. Show that this defines an inner product space on $V_2(\mathbb{F})$. (CO2) 10

6. Answer any one of the following:-

- 6-a. If $u = \frac{x^2}{a^2 + u} + \frac{y^2}{b^2 + u} + \frac{z^2}{z^2 + u}$, prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}\right]$. (CO3) 10
- 6-b. If $y = \sin(a \sin^{-1}x)$, then find $y_n(0)$. (CO3) 10

7. Answer any one of the following:-

- 7-a. In estimating the number of bricks in a pile which is measured to be (5 m x 10 m x 5 m) the count of bricks is taken as 100 bricks/meter³. Find the error in the cost when the tape is stretched 2% beyond its standard length. The cost of bricks is Rs. 2000 per thousand bricks. (CO4) 10
- 7-b. Expand $e^{x \cos y}$ in the powers of $(x - 1)$ and $\left(y - \frac{\pi}{4}\right)$ upto the third degree terms. (CO4) 10

8. Answer any one of the following:-

- 8-a. (i) If in certain code 1326 is coded as 8673, and 5670 is coded as 4329, then the code for 0009 10

will be

(ii) The total population of a village is 5000. The number of male and female increases by 10% and 15% respectively and consequently the population of the village become 5600. What was the number of males in the village?

(iii) A dealer offers a discount of 10% on the marked price of an article and still makes a profit of 20%. If its marked price is Rs. 800, then the cost price is? (CO5)

8-b. (i) A shopkeeper allows a 10% discount of to his customers and still gains 20%. Find the marked price of the article which costs Rs 450. 10

(ii) The average of marks of 17 students in an examination was calculated as 71. But it was later found that the mark of one student had been wrongly entered as 65 instead of 56 and another as 24 instead of 50. The correct average is

(iii) If the numerator of a fraction is increased by 20% and its denominator is decreased by 10%, the fraction becomes $\frac{3}{2}$. Find the original fraction. (CO5)