Printed page:02 **Subject Code: AAS0103**

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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B. Tech.

(SEM: FIRST SEMESTER THEORY EXAMINATION (2020-2021))

Subject Name: Engineering Mathematics-I

Time: 3 Hours **Max. Marks:100**

General Instructions:

- All questions are compulsory. Answers should be brief and to the point.
- This Question paper consists of 02 pages & 8 questions.
- It comprises of three Sections, A, B, and C. You are to attempt all the sections.
- Section A -Question No- 1 is objective type questions carrying 1 mark each, Question No- 2 is very short answer type carrying 2 mark each. You are expected to answer them as directed.
- Section B - Question No-3 is Long answer type -I questions with external choice carrying 6 marks each. You need to attempt any five out of seven questions given.
- Section C -Question No. 4-8 are Long answer type –II (within unit choice) questions carrying 10marks each. You need to attempt any one part *a* or *b*.
- Students are instructed to cross the blank sheets before handing over the answer sheet to the invigilator.
- No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION – A

1.	**Attempt all the parts.**	**[10×1=10]**	**CO**
a.	A is a singular matrix of order 3 with eigen values 2 and 3. The third eigen value is (a) 1 (b) 0 (c) 4 (d) -1	**(1)**	**CO1**
b.	The rank of the matrix $\begin{bmatrix}1 & 1 & 1\\1 & -1 & 0\\1 & 1 & 1\end{bmatrix}$ is (a) 0 (b) 1 (c) 2 (d) 3	**(1)**	**CO1**
c.	If $u = \frac{x^2}{a} + \frac{y^2}{b} - 7$ then $\frac{\partial u}{\partial x}$ is	**(1)**	**CO2**
d.	If $u = x^2$ and $x = t^3$ then $\frac{du}{dt}$ is	**(1)**	**CO2**
e.	If $x = rcos\emptyset$ and $y = rsin\emptyset$ then $\frac{\partial(x,y)}{\partial(r,\emptyset)}$ is	**(1)**	**CO3**
f.	The function $z = y^2 + x^2y + x^4$ has a minimum at $(0,0)$. (T/F)	**(1)**	**CO3**
g.	The value of the double integral $\int_{x=0}^{3}\int_{y=0}^{1}(x^2 + 3y^2)dydx$ is 12. (T/F)	**(1)**	**CO4**
h.	The value of $\int_0^\infty e^{-x^2}dx$ is $\sqrt{\pi}$. (T/F)	**(1)**	**CO4**
i.	The value of $81 \times 81 + 68 \times 68 - 2 \times 81 \times 68$ is	**(1)**	**CO5**
j.	Insert the missing number: 11, 13, 17, 19, 23, 29, 31, 37, 41, (.....).	**(1)**	**CO5**
2.	**Attempt all the parts.**	**[5×2=10]**	**CO**
a.	Find a and b such that $A = \begin{bmatrix}a & 4\\1 & b\end{bmatrix}$ has 3 and -2 as eigen values.	**(2)**	**CO1**
b.	Find the n^{th} derivative of $y = sin\,(ax + b)$.	**(2)**	**CO2**
c.	The radius of a sphere is found to be 10 meter with a possible error of 0.02 meter. What is the relative error in calculating the volume of sphere?	**(2)**	**CO3**
d.	Prove that Beta function is symmetric.	**(2)**	**CO4**
e.	If 50% of $(x - y)$ is 30% of $(x + y)$ then what percent of x is y?	**(2)**	**CO5**

SECTION – B

3. Answer any five of the following- **[5×6=30]** **CO**

a. Show that the system of equations **(6)** **CO1**

$$\left.\begin{aligned} 3x + 4y + 5z &= \alpha \\ 4x + 5y + 6z &= \beta \\ 5x + 6y + 7z &= \gamma \end{aligned}\right\}$$

is consistent only if α, β and γ are in arithmetic progression.

b. Trace the curve $a^2y^2 = x^2(a^2 - x^2)$. **(6)** **CO2**

c. Prove that $\frac{1}{(1-x)} = \frac{1}{3} + \frac{(x+2)}{3^2} + \frac{(x+2)^2}{3^3} + \frac{(x+2)^3}{3^4} + \cdots$ .... **(6)** **CO3**

d. Change the order of integration and hence evaluate $\int_0^a \int_{x^2/a}^{2a-x} xy\, dydx$. **(6)** **CO4**

e. Using the transformation $x + y = u$ and $y = uv$, show that $\int_0^1 \int_0^{1-x} e^{\left(\frac{y}{x+y}\right)} dydx = \frac{1}{2}(e - 1)$. **(6)** **CO4**

f. The selling price of 20 articles is equal to the cost price of 25 articles. Find the profit percent. **(6)** **CO5**

g. If the word LEADER is coded as 20-13-9-12-13-26, how would you write LIGHT? **(6)** **CO5**

SECTION – C

4 Answer any one of the following- **[5×10=50]** **CO**

a. State Cayley-Hamilton theorem. Verify Cayley-Hamilton theorem for the matrix **(10)** **CO1**

$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1} .

b. Find eigen values and corresponding eigen vectors of the matrix **(10)** **CO1**

$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$.

5. Answer any one of the following-

a. If $y = x^n logx$, prove that $(i)\ y_{n+1} = \frac{n!}{x}$ $(ii)\ x^2y_{p+2} + (2p - 2n + 1)xy_{p+1} + (p - n)^2y_p = 0$. **(10)** **CO2**

b. State and prove Euler's theorem for homogeneous function. Also prove that if $u = tan^{-1}\left[\frac{x^3+y^3}{x-y}\right]$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$. **(10)** **CO2**

6. Answer any one of the following-

a. Expand x^y in powers of $(x - 1)$ and $(y - 1)$ upto the third degree terms. **(10)** **CO3**

b. Find a point on the paraboloid $z = x^2 + y^2$ nearest to the point (3,-6,4). **(10)** **CO3**

7. Answer any one of the following-

a. Prove by the method of double integration that the area lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. **(10)** **CO4**

b. Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ by using Dirichlet's theorem. **(10)** **CO4**

8. Answer any one of the following-

a. A batsman makes a score of 87 runs in the 17^{th} inning and thus increases his average by 3. Find his average after 17^{th} inning. **(10)** **CO5**

b. If three numbers are added in pairs, the sums equal 10, 19 and 21. Find the numbers. **(10)** **CO5**