**Printed page: 3** **Subject Code: AAS0301B**

**Roll No:** | | | | | | | | | | | | | |

## NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY ,GREATER NOIDA

**(An Autonomous Institute Affiliated to AKTU, Lucknow)**

**B.Tech.**
**(SEM: III THEORY EXAMINATION (2021-2022))**
**Subject Name: Engineering Mathematics-III**

**Time: 3Hours** **Max. Marks:100**

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**General Instructions:**

- All questions are compulsory. It comprises of three Sections, A, B, and C.
- Section A -Question No- 1 is objective type questions carrying 1 mark each, Question No- 2 is very short answer type carrying 2 mark each.
- Section B - Question No-3 is Long answer type -I question with external choice carrying 6 marks each.
- Section C -Question No. 4-8 are Long answer type –II (within unit choice) questions carrying 10marks **each**.
- No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

| | | **SECTION – A** | | |
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| | | | | |
| **1.** | **All questions are compulsory-** | | **[10×1=10]** | **CO** |
| | **a.** | An analytic function with constant imaginary part is ........ | **(1)** | **CO1** |
| | **b.** | If $2x - x^2 + ay^2$ is to be harmonic, then $a$ should be<br>(a) 1<br>(b) 2<br>(c) 3<br>(d) 0 | **(1)** | **CO1** |
| | **c.** | The value of $\oint cosz\, dz$ along the closed curve $C$, where $C: \|Z\| = 1$, is ......... | **(1)** | **CO2** |
| | **d.** | The singularity $z = 0$ of $f(z) = \frac{sinz}{z}$ is<br>(a) pole<br>(b) essential singularity<br>(c) removable singularity<br>(d) none of these | **(1)** | **CO2** |
| | **e.** | The equation $\left[\frac{\partial^2 z}{\partial x^2} + 2xy\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial z}{\partial y} = 5x\right]$ is of order ...... and degree....... | **(1)** | **CO3** |
| | **f.** | The general solution $u(x, y)$ of the partial differential equation $4u_{xx} - u_{yy} = 0$ is<br>(a) $f(x) + g(y)$<br>(b) $f(x + 2y) + g(x - 2y)$<br>(c) $f(x + 4y) + g(x - 4y)$<br>(d) $f(4x + y) + g(4x - y)$ | **(1)** | **CO3** |
| | **g.** | Convolution theorem for Fourier transforms states that .......... | **(1)** | **CO4** |

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| | h. | The value of $Z\{1\}$, where $Z$ is the z-transform operator, is<br>(a) 1<br>(b) $\frac{1}{z-1}$<br>(c) $\frac{z}{z-1}$<br>(d)None of these | **(1)** | **CO4** |
| | i. | A pipe can fill a tank in 5 hours. The part of tank filled in two hours is...... | **(1)** | **CO5** |
| | j. | A, B, C, D, E and F, not necessarily in that order, are sitting at a round table. A is between D and F, C is opposite to D and D and E are not on neighbouring chairs. Which one of the following pairs must be sitting on neighbouring chairs?<br>(a) A and B<br>(b) C and E<br>(c) B and F<br>(d) A and C | **(1)** | **CO5** |
| **2.** | **All questions are compulsory-** | | **[5×2=10]** | **CO** |
| | a. | Prove that the function $f(z) = sinhz$ is analytic. | **(2)** | **CO1** |
| | b. | Explain Cauchy- Goursat theorem with example. | **(2)** | **CO2** |
| | c. | Classify the following partial differential equation<br>$$2\left(\frac{\partial^2 u}{\partial x^2}\right) + 4\left(\frac{\partial^2 u}{\partial x \partial y}\right) + 3\left(\frac{\partial^2 u}{\partial y^2}\right) = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + u$$ | **(2)** | **CO3** |
| | d. | Find the Fourier sine transform of $e^{-\lvert x \rvert}$ . | **(2)** | **CO4** |
| | e. | At what time between 5 and 6 O'clock will the hands of a clock be at right angle? | **(2)** | **CO5** |
| **<u>SECTION – B</u>** | | | | |
| | | | | |
| **3.** | **Answer any <u>five</u> of the following-** | | **[5×6=30]** | **CO** |
| | a. | Discuss the analyticity of the function $f(z) = Re(z^3)$ in the complex plane. | **(6)** | **CO1** |
| | b. | State and prove Cauchy Residue Theorem. | **(6)** | **CO2** |
| | c. | Find the solution of the partial differential equation $\left[\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u\right]$, where $u(x,0) = 6e^{-3x}$ using method of separation of variables. | **(6)** | **CO3** |
| | d. | Find the Fourier cosine transform of $F(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$ | **(6)** | **CO4** |
| | e. | A, B, and C can complete a piece of work in 10, 15 and 18 days. In how many days, would all of them complete the same work working together? | **(6)** | **CO5** |
| | f. | State and prove Liouvilles's Theorem. | **(6)** | **CO2** |
| | g. | A man rows at a speed of 8 Km/h in still water to a certain distance upstream and back to the starting point in a river which flows at 4 Km/h. Find his average speed for total journey. | **(6)** | **CO5** |

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| **SECTION – C** | | | | |
| 4 | **Answer any one of the following-** | | **[5×10=50]** | **CO** |
| | **a.** | If $w = f(z) = u + iv$ is an analytic function and $u - v = e^x(cosy - siny)$ then find analytic function $w$ in terms of $z$. | **(10)** | **CO1** |
| | | | | |
| | **b.** | Show that the transformation $w = \left(\frac{5-4z}{4z-2}\right)$ transforms the circle $\|z\| = 1$ into a circle of radius unity in $w$-plane and find the centre of the circle. | **(10)** | **CO1** |
| **5.** | **Answer any one of the following-** | | | |
| | **a.** | By the method of contour integration, prove that $\int_{-\infty}^{\infty} \frac{sinmx}{a^2+x^2} dx = 0, m > 0, a > 0.$ | **(10)** | **CO2** |
| | | | | |
| | **b.** | Obtain the series which represents the function $\left[\frac{z^2-1}{(z+2)(z+3)}\right]$ in the regions<br>(a) $\|z\| < 2$<br>(b) $2 < \|z\| < 3$<br>(c) $\|z\| > 3$ | **(10)** | **CO2** |
| **6.** | **Answer any one of the following-** | | | |
| | **a.** | Use the method of separation of variables to solve the equation $\frac{\partial\omega}{\partial t} = \frac{\partial^2\omega}{\partial x^2}$ . Given that $\omega = 0$ when $t \to \infty$ as well as $\omega = 0$ at $x = 0$ and $x = L$. | **(10)** | **CO3** |
| | | | | |
| | **b.** | Solve the following partial differential equation<br>$(D^2 - 3DD' + 2D'^2)z = e^{2x-y} + e^{x+y} + cos(x + 2y)$ | **(10)** | **CO3** |
| **7.** | **Answer any one of the following-** | | | |
| | **a.** | Using Fourier transform, solve the following equation<br>$\frac{\partial u}{\partial x} = c^2 \frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty, t > 0$<br>with $u(x, 0) = \begin{cases} 1, -1 < x < 0 \\ -1,\ 0 < x < 1 \\ 0, otherwise \end{cases}$ | **(10)** | **CO4** |
| | | | | |
| | **b.** | Using the Z-transform, solve $y_{k+2} - 2y_{k+1} + y_k = 3k + 5.$ | **(10)** | **CO4** |
| | | | | |
| **8.** | **Answer any one of the following-** | | | |
| | **a.** | What day of the week was on June 5, 1999? | **(10)** | **CO5** |
| | | | | |
| | **b.** | Two taps A and B can fill a cistern in 30 minutes and 60 minutes, respectively. There is a third exhaust tap C at the bottom of the tank. If all the taps are opened at the same time, the cistern will be full in 45 minutes. In what time can exhaust tap C empty the cistern when it is full? | **(10)** | **CO5** |