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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech.

SEM: II - CARRY OVER THEORY EXAMINATION - JUNE (2021 - 2022)

Subject: Engineering Mathematics-II

Time: 3 Hours

Max. Marks: 100

General Instructions:

1. The question paper comprises three sections, A, B, and C. You are expected to answer them as directed.
2. Section A - Question No- 1 is 1 marker & Question No- 2 carries 2 mark each.
3. Section B - Question No-3 is based on external choice carrying 6 marks each.
4. Section C - Questions No. 4-8 are within unit choice questions carrying 10 marks each.
5. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION A

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1. Attempt all parts:-

- 1-a. Degree and order of the differential equation $\sqrt{((dy/dx)^2 + 3y)} = (d^2y)/(dx^2)$ is (CO1) 1
- (a) Ord = 2, Deg = 2
 - (b) Ord = 2, Deg = 1
 - (c) Ord = 1, Deg = 2
 - (d) Ord = 1, Deg = 1
- 1-b. The value for $\frac{1}{D-2} \sin 2x$ (CO1) 1
- (a) $e^{2x} \int e^{2x} \sin 2x dx$
 - (b) $e^{2x} \int e^{-2x} \sin 2x dx$
 - (c) $e^{-2x} \int e^{2x} \sin 2x dx$
 - (d) None of these
- 1-c. The coefficient a_0 in a Fourier series for the function $f(x) = x^2$ in the interval $-1 < x < 1$ is (CO2) 1
- (a) $\frac{1}{\pi}$
 - (b) $\frac{2\pi^2}{3}$
 - (c) 0
 - (d) none of these
- 1-d. The auxiliary series for the comparison test to the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n + \sqrt{n}}$ is (CO2) 1
- (a) $\frac{1}{\sqrt{n}}$
 - (b) $\frac{1}{n}$
 - (c) $\frac{1}{n^3}$
 - (d) none of these
- 1-e. Laplace transform of $tu_2(t)$ is (CO3) 1

(a) $\left(\frac{1}{s^2} + \frac{2}{s}\right)e^{-2s}$

(b) $\left(\frac{1}{s^2} - \frac{2}{s}\right)e^{-2s}$

(c) $\frac{1}{s^2}e^{-2s}$

(d) $\frac{1}{s}e^{-2s}$

1-f. Inverse Laplace of the function $f(s) = \frac{1}{(s+1)^2}$ is (CO3) 1

(a) te^t

$\frac{e^{-t}}{t}$

(b) t

(c) te^{-t}

(d) None of these

1-g. Given that $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, which of the following statements is false? (CO4) 1

(a) $\nabla(r^n) = nr^{n-2}\vec{r}$

(b) $\nabla\left(\frac{1}{r}\right) = -\frac{1}{r^3}\vec{r}$

(c) $\nabla(\sin(r)) = \cos(r)\frac{\vec{r}}{r}$

(d) $\nabla(\log r) = \frac{\vec{r}}{r}$

1-h. By the use of Green's theorem, the area bounded by a simple closed curve C is given by (CO4) 1

(a) $\frac{1}{2} \int xdx + ydy$

(b) $\frac{1}{2} \int xdy + ydx$

(c) $\frac{1}{2} \int xdy - ydx$

(d) $\frac{1}{2} \int xdx - ydy$

1-i. A boy has coins in the denominations of ₹ 1 and ₹ 2. If he has total 30 coins and the value of coins is ₹ 48. Find the number of ₹ 1 coins he has. (CO5) 1

(a) 18

(b) 10

(c) 12

(d) 14

1-j. Pointing to a photograph of a boy Suresh said, "He is the son of the only son of my mother." How is Suresh related to that boy? (CO5) 1

(a) Brother

(b) Uncle

(c) Father

(d) Cousin

2. Attempt all parts:-

- 2.a. Find the differential equation of the family of curves $y = a e^{4x} + b e^{-x}$ where a and b are arbitrary constants. (CO1) 2
- 2.b. Find the Fourier coefficient a_n for $f(x) = x^2 - 2, -2 < x < 2$. (CO2) 2
- 2.c. Find Laplace transform of the function $F(t) = (t+2)^2 e^t$. (CO3) 2
- 2.d. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then show that $\text{grad } r^n = nr^{n-2}\vec{r}$, where $r = |\vec{r}|$. (CO4) 2
- 2.e. ₹ 385 were divided among P, Q and R in such a way that P had ₹ 20 more than Q and R had ₹ 15 more than P. How much was R's share? (CO5) 2

SECTION B

30

3. Answer any five of the following:-

- 3-a. Solve the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$. (CO1) 6
- 3-b. Solve $\frac{dx}{dt} - y = e^t, \frac{dy}{dt} - x = \sin t; x(0) = 1, y(0) = 0$. (CO1) 6
- 3-c. Obtain the Fourier series for $f(x) = \left(\frac{\pi - x}{2}\right), 0 < x < 2$. (CO2) 6
- 3-d. Test the convergence of the series, $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \frac{x^4}{4.5} + \dots$. (CO2) 6
- 3.e. Find the Laplace Transform of the function $F(t) = \int_0^t te^{-t} \sin 4t dt$. (CO3) 6
- 3.f. Show that the vector field $\vec{F} = \frac{\vec{r}}{r^3}$ is solenoidal. (CO4) 6
- 3.g. (i) 10 years ago, the ages of A and B were in the ratio of 13: 17. 17 years from now the ratio of their ages will be 10: 11. What is the age of B at present? 6
(ii) The ages of Nishi and Vinnee are in the ratio of 6:5 After 9 years the ratio of their ages will be 9:8 What is the difference in their ages? (CO5)

SECTION C

50

4. Answer any one of the following:-

- 4-a. Solve the following differential equation by changing the independent variable $\frac{d^2y}{dx^2} - \frac{1}{x} \frac{dy}{dx} + 4x^2y = x^4$. CO 1 10
- 4-b. Solve $\frac{dx}{dt} + 2x - 3y = t, \frac{dy}{dt} - 3x + 2y = e^{2t}$. CO 1 10

5. Answer any one of the following:-

- 5-a. Obtain the Fourier series to represent function $f(x) = x \sin x$ in the interval $-\pi \leq x \leq \pi$. (CO2) 10
Hence show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \dots = \frac{\pi - 2}{4}$ 10
- 5-b. Obtain the Fourier series for the function $f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$ (CO2) 10
Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

6. Answer any one of the following:-

- 6-a. State and prove the Convolution theorem of inverse Laplace transform. (CO3) 10
- 6-b. Solve the following differential equation by Laplace transform 10

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t} \sin t, \text{ Given that } x = 0, \frac{dx}{dt} = 1 \text{ at } t = 0. \quad (\text{CO3})$$

7. Answer any one of the following:-

7-a. Verify divergence theorem for $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ taken over the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. (CO4) 10

7-b. Verify Stokes theorem for the vector field $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy plane. (CO4) 10

8. Answer any one of the following:-

8-a. (i) Two numbers are in the ratio 3: 5. If 9 be subtracted from each, then they are in the ratio of 12:23. Find the second number? 10

(ii) Alloy A contains 40% gold and 60% silver. Alloy B contains 35% gold and 40% silver and 25% copper. Alloy A and B are mixed in the ratio of 1:4. What is the ratio of gold and silver in the newly formed alloy? (CO5)

8-b. (i) Study the information given below and answer the following questions: Every morning Tanisha goes for cycling and follows a fixed route. From her hostel, she goes in the north direction and cycles 20 km. She then takes a right turn and moves 30 km, followed by a right turn again and cycles for 35 km. She then turns left and cycles for 15 km. Finally, she takes a left turn and cycles for 15 km . 10

(a) In which direction is Tanisha from her hostel, when she reaches the final destination?

(b) How far is the final destination from Tanisha's hostel?

(ii) Ashish has to go to his coaching class 5 days in a week. He walks to the Institute all by himself. Starting from his house, he starts moving East and walks 90 m. He then turned right and walked 20 m. He then took a right turn and walked for 30 m. From there, Ashish moved 100 m to the north and reached his Coaching Institute. How far is his house from the coaching centre? (CO5)