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Subject Code:- AAS0301B

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: III - CARRY OVER THEORY EXAMINATION - AUGUST 2023

Subject: Engineering Mathematics-III

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* **three Sections -A, B, & C.** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION A **20**

1. Attempt all parts:-

1-a. If $u = x^2 - y^2$, then the conjugate harmonic function is (CO1) 1

(a) e^z

(b) $x^2 - y^2$

(c) $x^2 + y^2$

(d) 2xy

1-b. If f(z) is an entire function such that $f(z) \leq M$ for all $z \in C$, then f(z) is (CO1) 1

(a) Constant

(b) Non constant

(c) Exponential function

(d) None of these

1-c. The value of the integral $\int_C \frac{dz}{z-a}$ where C is the circle $|z-a| = r$ (CO2) 1

(a) $2\pi i$

(b) $\frac{\pi i}{2}$

(c) $-\pi i$

(d) None of these

1-d. Point $z = a$ is called a removable singular point of $f(z)$,if (CO2) 1

(a) $\lim_{z \to a} f(z)$ exist and non-zero.

(b) $\lim_{z \to a} f(z)$ exist and finite.

(c) $\lim_{z \to a} f(z)$ does not exist.

(d) None of these

1-e. Classification of pde: $u_{xx} + 2u_{xt} + u_{tt} = 0$ is: (CO3) 1

(a) Parabolic

(b) Hyperbolic

(c) Elliptic

(d) None of these

1-f. A solution to partial differential equation $\frac{\partial^2 u}{\partial x^2} - 9\frac{\partial^2 u}{\partial y^2} = 0$ is (CO3) 1

(a) $cos(3x - y)$

(b) $f_1(y+3x) + xf_2(y-3x)$

(c) $f_1(y+3x) + f_2(y-3x)$

(d) $f_1(y+3ix) + f_2(y-3ix)$

1-g. Order of the difference equation $y_{k+2} + y_{k+1} - y_k = 0$ is (CO4) 1

(a) 3

(b) 2

(c) 1

(d) 0

1-h. If $f_1(p)$ and $f_2(p)$ are the Fourier transforms of $F_1(x)$ and $F_2(x)$ respectively then $F[c_1F_1(x) + c_2F_2(x)]$ (CO4) 1

(a) $c_1f_1(p) + c_2f_2(p)$

(b) $f_1(p) + f_2(p)$

(c) $c_1f_1(p_1) + c_2f_2(p_2)$

(d) None of these

1-i. If a person walks at 14 km/hr instead of 10 km/hr, he would have walked 20 km more. The actual distance travelled by him is: (CO5) 1

(a) 50 km

(b) 56 km

(c) 70 km

(d) None of these

1-j. P and Q together can do a work in 18 days. P alone can do the same work in 27 days. In how many days can Q alone do the same work? (CO5) 1

(a) 54 days

(b) 36 days

(c) 45 days

(d) None of these

2. Attempt all parts:-

2.a. Write Cauchy Riemann equation for cartesian coordinates. (CO1) 2

2.b. Evaluate: $\oint_C \frac{dz}{z^2+9}$; where $C:|z-3i|=4$.(CO2) 2

2.c. Solve r- 4s+4t=0 (CO3) 2

2.d. Write shifting property for Fourier transform. (CO4) 2

2.e. A, P, R, X, S and Z are sitting in a row. S and Z are in the Centre. A and P are at the ends. R is sitting to the left of A. Who is to the right of P? (CO5) 2

SECTION B **30**

3. Answer any <u>five</u> of the following:-

3-a. Find the bilinear transformation which maps the points $z=1,-1,i$ into the points $w=0,1,\infty$ respectively. (CO1) 6

3-b. Determine an analytic function $f(z)$ in terms of z whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$. (CO1) 6

3-c. Evaluate $\int_0^{\pi} \frac{(1+2\cos\theta)d\theta}{5+4\cos\theta}$ (CO2) 6

3-d. Discuss the nature of singularity of $f(z)=\frac{z-\sin z}{z^3}$ at z=0 (CO2) 6

3.e. Solve $(D^2 + 3DD'+2D'^2)z = x + y$. (CO3) 6

3.f. Find the Fourier cosin transform of e^{-x^2}. (CO4) 6

3.g. Prove that the calendar for the year 2003 will serve for the year 2014.(CO5) 6

SECTION C **50**

4. Answer any <u>one</u> of the following:-

4-a. Show that $e^x \cos y$ is a harmonic function, find the analytic function of which it is real part. (CO1) 10

4-b. Prove that the function $f(z)=\left\{\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2},\quad z\neq 0\right\},\ f(0)=0,$ satisfy C-R equation at origin but f(z) is not analytic at origin. (CO1) 10

5. Answer any <u>one</u> of the following:-

5-a. Expand $f(z)=\frac{z}{(z-1)(2-z)}$ 10

(i) $0<|z-2|<1$

(ii) $|z-1|>1$ (CO2)

5-b. Verify the Cauchy theorem by integrating e^{iz} along the boundary of the triangle with the vertices at the points $1+i$, $-1+i$ and $-1-i$. (CO2) 10

6. Answer any <u>one</u> of the following:-

6-a. Solve by the method of separation of variables: $\frac{\partial u}{\partial x}+4\frac{\partial u}{\partial t}=3u,\ u=3e^{-x}-e^{-5x}$, when t=0. (CO3) 10

6-b. Solve the PDE: $(D^2-DD'+D'-1)z=e^y+\cos(x+2y)$. (CO3) 10

7. Answer any <u>one</u> of the following:-

7-a. Find the Fourier sine transform of $\frac{e^{-ax}}{x}, a>0$. Hence find Fourier sine transform of $\frac{1}{x}$. (CO4) 10

7-b. Solve by z- transform the difference equation $y_{k+2}+6y_{k+1}+9y_k=2^k$; $y_0=y_1=0$. (CO4) 10

8. Answer any <u>one</u> of the following:-

8-a. **(i)** Three pipes A, B and C are attached to a tank. A and B can fill it in 20 and 30 minutes respectively while C can empty it in 15 minutes. If A, B and C are kept open successively for 1 minute each, how soon will the tank be filled? 10

(ii) A cistern has three pipes A, B and C. A and B can fill it in 3 hours and 4 hours respectively while C can empty the completely filled cistern in 1 hour. If the pipes are opened in order at 3, 4 and 5 p.m. respectively, at what time will the cistern be empty? (CO5)

8-b. At what time between 4 and 5 o'clock will the hands of a clock be at right angle? (CO5) 10