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**NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY ,GREATER NOIDA**  
**(An Autonomous Institute Affiliated to AKTU, Lucknow)**

**B.Tech**

**(SEM:I THEORY EXAMINATION(2020-2021))**

**Subject Name: Discrete Mathematics**

**Time: 3Hours**

**Max. Marks:100**

**General Instructions:**

- All questions are compulsory. Answers should be brief and to the point.
- This Question paper consists of 3 pages & 8 questions.
- It comprises of three Sections, A, B, and C. You are to attempt all the sections.
- **Section A** -Question No- 1 is objective type questions carrying 1 mark each, Question No- 2 is very short answer type carrying 2 mark each. You are expected to answer them as directed.
- **Section B** - Question No-3 is Long answer type -I questions with external choice carrying 6 marks each. You need to attempt any five out of seven questions given.
- **Section C** -Question No. 4-8 are Long answer type –II (within unit choice) questions carrying 10marks each. You need to attempt any one part *a* or *b*.
- Students are instructed to cross the blank sheets before handing over the answer sheet to the invigilator.
- No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

**SECTION – A**

- |           |  |                  |            |
|-----------|--|------------------|------------|
| <b>1.</b> | <b>Answer all the parts-</b>   | <b>[10×1=10]</b> |            |
|           | <b>a.</b> How many bytes are required to encode 2000 bits of data?   | (1)              | <b>CO1</b> |
|           | <b>b.</b> What is De-Morgan's law?   | (1)              | <b>CO2</b> |
|           | <b>c.</b> If A has 4 elements B has 8 elements then find the minimum and maximum number of elements in A U B.  | (1)              | <b>CO2</b> |
|           | <b>d.</b> How many NAND gates are required to make an XOR gate?  | (1)              | <b>CO1</b> |
|           | <b>e.</b> A drawer contains 12 red and 12 blue socks, all unmatched. A person takes socks out at random in the dark. How many socks must he take out to be sure that he has at least two blue socks? | (1)              | <b>CO3</b> |
|           | <b>f.</b> What are the minimum numbers of edges in a connected cyclic graph on n vertices?   | (1)              | <b>CO4</b> |
|           | <b>g.</b> What is the Worst case complexity of Breadth First Search traversal?   | (1)              | <b>CO4</b> |
|           | <b>h.</b> Describe the Boolean duality principle.  | (1)              | <b>CO1</b> |
|           | <b>i.</b> Define the term chromatic number.  | (1)              | <b>CO4</b> |
|           | <b>j.</b> Define SOP and POS.  | (1)              | <b>CO1</b> |
| <b>2.</b> | <b>Answer all the parts-</b>   | <b>[5×2=10]</b>  | <b>CO</b>  |
|           | <b>a.</b> Define pigeon hole principle.  | (2)              | <b>CO3</b> |
|           | <b>b.</b> Show that every cyclic group is abelian.   | (2)              | <b>CO2</b> |
|           | <b>c.</b> Define Ring and Field, also give example of each.  | (2)              | <b>CO2</b> |
|           | <b>d.</b> Show that following are tautologies :<br>(i) $(p \wedge (p \rightarrow q)) \rightarrow q$ (ii) $(p \vee \neg(p \wedge q))$   | (2)              | <b>CO5</b> |
|           | <b>e.</b> Let Z be the group of integers with binary operation * defined by $a * b = a + b - 2$ , for all $a, b, \in Z$ . Evaluate the identity element and inverse element of the group $(Z, *)$ .  | (2)              | <b>CO2</b> |

**SECTION – B**

3. Answer any five of the following- [5×6=30] CO
- a. Simplify the Boolean function using Karnaugh Map— (6) CO1  
 $F(A,B,C,D) = \Sigma(0,2,5,8,9,10,11,12,13,14,15)$
- b. prove the following relations: (6) CO2  
 (i).  $(A \cup B)' = A' \cap B'$  (ii).  $(A \cap B)' = A' \cup B'$
- c. Show that the multiplicative group  $G = \{1, -1, i, -i\}$  is cyclic. Also find its generators. (6) CO2
- d. Prove by mathematical induction that  $n^3 + 2n$  is divisible by 3 for all positive integers n. (6) CO3
- e. Explain in detail the Euler's formula with the help of an example. (6) CO4
- f. Write the statement of the Four-color theorem. Also give an example of it (6) CO4
- g. In context to the Logics, Explain the following: (6)  
 i. truth assignments and truth tables CO5  
 ii. Soundness and completeness

**SECTION – C**

- 4 Answer any one of the following- [5×10=50] CO
- a. Design logic circuits, using AND, OR, and NOT gates to solve the following problems. (10) CO1  
 i. Input two bits, x, y and output two bits representing  $x-y$  ( $1-1 = 00, 1-0 = 01, 0-0 = 00, 0-1 = 11$ ).  
 ii. Input three bits x, y, z and output one bit which is the majority of the three input bits
- b. Describe the Boolean duality principle. Write the dual of each Boolean equations: (10) CO1  
 i.  $x+x'y=x+y$   
 ii.  $(x.1)+(x'+0)=0$
5. Answer any one of the following-
- a. Let  $A = \{1, 2, 4, 5, 7, 8\}$ ,  $B = \{x|(x \in Z) \wedge (x < 10)\}$ ,  $C = \{x|(x \in Z) \wedge (x \bmod 3 < 2)\}$ . Assume the universe is Z. (10) CO2  
 Prove the following set identities, using either Venn Diagrams or the rules of sets.  $A \cap (B - A) = \emptyset$   
 $(A - C) \cap (C - B) = \emptyset$
- b. Prove that the set  $G = \{1,2,3,4,5,6\}$  under the multiplicative modulo 7 is a group. Find the orders and subgroup generated by 2 and 3. Is it cyclic group? (10) CO4
6. Answer any one of the following-
- a. Solve  $a_r - 6a_{r-1} + 8a_{r-2} = 0$ , given  $a_0=8$  and  $a_1=1$ , Explain the role of combinatronics in discrete structure. (10) CO3
- b. Each of 15 red balls and 15 green balls is marked with an integer between 1 and 100 inclusive; no integer appears on more than one ball. The value of a pair of balls is the sum of the numbers on the balls. Show there are at least two pairs, consisting of one red and one green ball, with the same value. Show that this is not necessarily true if there are 13 balls of each color. (10) CO3

7. Answer any one of the following-

- a. Explain why a graph with  $n$  vertices, where each vertex has degree at least  $n - 2$ , cannot be bipartite when  $n \geq 5$  (10) CO4
- b. Briefly explain on the followings: (10) CO4
- i. subgraph,
  - ii. spanning graph,
  - iii. complement of graph,
  - iv. self-complementary graph,
  - v. induced subgraph

8. Answer any one of the following-

- a. Write the following statements using quantifiers and logical connectives (10) CO5
- i) Mathematics books that published in India has a blue cover.
  - ii) There exists a mathematics book with a cover that is not blue.
  - iii) There are mathematics books that are published outside India
  - iv) Not all books have bibliographies.
- He eat crackers only if he drinks milk.
- b. Define quantifiers in predicate logic? Test the validity of the following arguments: “All educated persons are well behaved. Ram is educated. No well-behaved person is quarrelsome. Therefore, Ram is not quarrelsome” (10) CO5