Printed Page:-5 **Subject Code:- CMIAS0103**
Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA
(An Autonomous Institute Affiliated to AKTU, Lucknow)
M.Tech Integrated
SEM: I - THEORY EXAMINATION (2025 - 2026)
Subject: Calculus and Linear Algebra

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* ***three Sections -A, B, & C.*** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

<u>SECTION-A</u> 20

1. Attempt all parts:-

1-a. If A is a Hermitian matrix, then iA is (CO1, K1) 1

- (a) Hermitian matrix
- (b) skew-Hermitian
- (c) Symmetric
- (d) none of these

1-b. If λ is a characteristic root of the matrix A, then a characteristic root of the matrix A+kI is (CO1, K1) 1

- (a) λ
- (b) λ+k
- (c) k-λ
- (d) none of these

1-c. The nth derivative of $\cos(ax+b)$ is (CO2, K1) 1

- (a) $a^n \cos(ax+b)$
- (b) $a^n \cos\left(ax+b+\frac{n\pi}{2}\right)$
- (c) $a^n \cos\left(ax+b+\frac{n\pi}{4}\right)$
- (d) None of these

1-d. The degree of homogeneous function $u(x,y) = x^{\frac{1}{3}} y^{\frac{-4}{3}} \sin^{-1}\left(\frac{y}{x}\right)$ is (CO2,K1) 1

- (a) -1

(b) 1
(c) 4
(d) -4/9

1-e. Stationary points of the function $f(x) = y^2 + 4xy + 3x^2 + x^3$ is (CO3,K1) 1

(a) $\left(1, -\frac{4}{3}\right)$
(b) $\left(\frac{2}{3}, -\frac{4}{3}\right)$
(c) $\left(\frac{2}{3}, -\frac{2}{3}\right)$
(d) $\left(\frac{2}{4}, -\frac{4}{3}\right)$

1-f. The expansion of sin x is (CO3,K1) 1

(a) $1 - x - \frac{x^2}{2!} - \frac{x^3}{3!} - \frac{x^4}{4!} + \ldots\ldots.$
(b) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \ldots\ldots.$
(c) $-x - \frac{x^3}{3!} - \frac{x^5}{5!} - \frac{x^7}{7!} \ldots\ldots.$
(d) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} \ldots\ldots.$

1-g. The value of $\beta(n,m) - \beta(m,n)$ is (CO4,K1) 1

(a) $\beta(m,n)$
(b) $2\beta(m,n)$
(c) 0
(d) $\beta(n,m)$

1-h. The value of integral $\int_0^5 \int_{2-x}^{2+x} dydx,$ is (CO4,K1) 1

(a) 21
(b) 25
(c) 20
(d) None of these

1-i. If $\varnothing = 3x^2y - y^3z^2$ then grad $\varnothing$ at the point $(1, -2, -1)$ is (CO5,K2) 1

(a) $-12\hat{i} - 9\hat{j} - 16\hat{k}$
(b) $12\hat{i} + 9\hat{j} + 16\hat{k}$
(c) $\hat{i} + \hat{j} + \hat{k}$
(d) $\hat{i} + \hat{j}$

1-j. **Stoke's Theorem is** (CO5 K1) 1

(a) $\int\int_S F.\hat{n}\, dS = \int_C \vec{F}.d\hat{r}$

(b) $\iint_S \text{Curl}\ \vec{F}.\hat{n}\ dS = \oint_C \vec{F}.d\hat{r}$

(c) $\iint_S \vec{F}.\hat{n}\ dS = \iiint_V \vec{F}.d\hat{r}$

(d) $\iint_S \vec{F}.\hat{n}\ dS = \iiint_V \text{div}\vec{F}\ dV$

2. Attempt all parts:-

2.a. Find the inverse by using elementary transformation where $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$. (CO1,K2) 2

2.b. Find the first order partial derivatives of $u = \log(x^2 + y^2)$ (CO2, K2) 2

2.c. If $u = x - y,\ v = 2y + 3z, w = 5z + 2x,$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. (CO3,k2) 2

2.d. Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\text{Cot}\ \theta}\ d\theta = \frac{\pi}{\sqrt{2}}$. (CO4,K3) 2

2.e. **Find the unit vector normal to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$.** (CO5 K2) 2

SECTION-B 30

3. Attempt all parts:-

3.a. Answer any <u>one</u> of the following:-

3.a.(i) Determine the value of λ and μ such that the system 6
$2x - 5y + 2z = 8,\ 2x + 4y + 6z = 5,\ x + 2y + \lambda z = \mu$
has (i) no solution (ii) a unique solution (iii) infinite number of solutions.(CO1,K2)

3.a.(ii) Find the rank of matrix $\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$ by reducing it to Normal form. (CO1,K3) 6

3.b. Answer any one of the following:-

3.b.(i) If $u = \log(\tan x + \tan y + \tan z)$, prove that $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$. (CO2,K3) 6

3.b.(ii) If $u = \sec^{-1}\left(\frac{x^3 - y^3}{x + y}\right)$ then evaluate $x^2 \frac{\partial^2 u}{\partial^2 x} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial^2 y}$. (CO2,K3) 6

3.c. Answer any one of the following:-

3.c.(i) A balloon is in the form of a right circular cylinder of radius 1.5 m and length 4m, which is surmounted by hemispherical ends. If the radius is increased by 0.01 m and the length by 0.05 m. Find the percentage change in the volume of balloon. (CO3, K3) 6

3.c.(ii) Expand $e^x \cos y$ in powers of x and y up to third degree term. (CO3, K2) 6

3.d. Answer any one of the following:-

3.d.(i) Change into polar co-ordinates and hence evaluate the integral 6

$$\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dy\,dx$$. (CO4,K2)

3.d.(ii) Using Beta and Gamma functions, evaluate $\int_0^1 \left(\frac{x^3}{1-x^3}\right)^{1/2} dx.$ (CO4,K3) 6

3.e. Answer any one of the following:-

3.e.(i) Show that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find the velocity potential $\varnothing$ such that $\vec{A} = \nabla \varnothing$. (CO5, K3) 6

3.e.(ii) Find the directional derivative of $f = 2x^2 + 3y^3 + z^2$ at the point $P(3,1,2)$ in the direction of the vector $a = \hat{i} - 2\hat{k}$. (CO5, K3) 6

SECTION-C 50

4. Answer any <u>one</u> of the following:-

4-a. Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. Hence compute A^{-1}. Also find the matrix represented by $A^8 - 11A^7 - 4A^6 + A^5 + A^4 - 11A^3 - 3A^2 + 2A + I$. (CO1,K2) 10

4-b. Find the eigen values and corresponding eigenvectors of a matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. (CO1,K3) 10

5. Answer any <u>one</u> of the following:-

5-a. If $u = f(r, s, t)$ and $r = \frac{x}{y}, s = \frac{y}{z}, t = \frac{z}{x}$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0.$ (CO2,K2) 10

5-b. If $y = (\sin^{-1}x)^2$ then prove that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$. Also find the value of y_n at $x = 0$. (CO2, K3) 10

6. Answer any <u>one</u> of the following:-

6-a. If u, v, w are the roots of the cubic equation $(x-a)^3 + (x-b)^3 + (x-c)^3 = 0$ in x, then find $\frac{\partial(u, v, w)}{\partial(a, b, c)}$. (CO3, K3) 10

6-b. Use the method of Lagrange's multiplier to find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. (CO3, K3) 10

7. Answer any <u>one</u> of the following:-

7-a. Evaluate by changing the order of integration $\int_0^a \int_{x^2/a}^{2a-x} xy\,dy\,dx$. (CO4,K2) 10

7-b. Apply Drichlet's integral to find the volume and mass cotained in the solid region in the first octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, if the density at any point is $\rho(x,y,z) = kxyz$ (CO4,K3) 10

8. Answer any <u>one</u> of the following:-

8-a. Verify Green's theorem in the plane for $\int_C 2y^2 dx + 3x\, dy$ where C is the boundary of the closed region bounded by $y = x$ and $y = x^2$. (CO5, K3) 10

8-b. Verify divergence theorem for $\vec{F} = (x3 - yz)\hat{i} - 2x2y\hat{j} + 2\hat{k}$ taken over the cube bounded by the planes $x = 0,\ x = a,\ y = 0,\ y = a,\ z = 0,\ z = a$. (CO5, K3) 10