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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA
(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: I - THEORY EXAMINATION (2025 - 2026)

Subject: Calculus and Linear Algebra

Time: 3 Hours

Max. Marks: 100

General Instructions:

IMP: Verify that you have received the question paper with the correct course, code, branch etc.

1. This Question paper comprises of **three Sections -A, B, & C**. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.

2. Maximum marks for each question are indicated on right -hand side of each question.

3. Illustrate your answers with neat sketches wherever necessary.

4. Assume suitable data if necessary.

5. Preferably, write the answers in sequential order.

6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION-A

20

1. Attempt all parts:-

1-a. If A is a Hermitian matrix, then iA is (CO1, K1)

1

- (a) Hermitian matrix
- (b) skew-Hermitian
- (c) Symmetric
- (d) none of these

1-b. If λ is a characteristic root of the matrix A, then a characteristic root of the matrix $A+kI$ is (CO1, K1)

1

- (a) λ
- (b) $\lambda+k$
- (c) $k-\lambda$
- (d) none of these

1-c. The n th derivative of $\cos(ax + b)$ is (CO2, K1)

1

- (a) $a^n \cos(ax + b)$
- (b) $a^n \cos\left(ax + b + \frac{n\pi}{2}\right)$
- (c) $a^n \cos\left(ax + b + \frac{n\pi}{4}\right)$
- (d) None of these

1-d. The degree of homogeneous function $u(x,y) = x^{\frac{1}{3}} y^{\frac{-4}{3}} \sin^{-1}\left(\frac{y}{x}\right)$ is (CO2,K1)

1

- (a) -1

- (b) 1
- (c) 4
- (d) -4/9

1-e. Stationary points of the function $f(x) = y^2 + 4xy + 3x^2 + x^3$ is (CO3,K1) 1

- (a) $\left(1, -\frac{4}{3}\right)$
- (b) $\left(\frac{2}{3}, -\frac{4}{3}\right)$
- (c) $\left(\frac{2}{3}, -\frac{2}{3}\right)$
- (d) $\left(\frac{2}{4}, -\frac{4}{3}\right)$

1-f. The expansion of $\sin x$ is (CO3,K1) 1

- (a) $1 - x - \frac{x^2}{2!} - \frac{x^3}{3!} - \frac{x^4}{4!} + \dots$
- (b) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$
- (c) $-x - \frac{x^3}{3!} - \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$
- (d) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$

1-g. The value of $\beta(n,m) - \beta(m,n)$ is (CO4,K1) 1

- (a) $\beta(m,n)$
- (b) $2\beta(m,n)$
- (c) 0
- (d) $\beta(n,m)$

1-h. The value of integral $\int_0^5 \int_{2-x}^{2+x} dy dx$, is (CO4,K1) 1

- (a) 21
- (b) 25
- (c) 20
- (d) None of these

1-i. If $\nabla \phi = 3x^2y - y^3z^2$ then grad ϕ at the point $(1, -2, -1)$ is (CO5,K2) 1

- (a) $-12\hat{i} - 9\hat{j} - 16\hat{k}$
- (b) $12\hat{i} + 9\hat{j} + 16\hat{k}$
- (c) $\hat{i} + \hat{j} + \hat{k}$
- (d) $\hat{i} + \hat{j}$

1-j. Stoke's Theorem is (CO5 K1) 1

- (a) $\int_S \mathbf{F} \cdot \hat{n} \, dS = \int_C \vec{F} \cdot d\vec{r}$

(b) $\int \int_S \text{Curl } \vec{F} \cdot \hat{n} \, dS = \oint_C \vec{F} \cdot d\vec{r}$
(c) $\int \int_S \vec{F} \cdot \hat{n} \, dS = \int \int \int_V \vec{F} \cdot d\vec{r}$
(d) $\int \int_S \vec{F} \cdot \hat{n} \, dS = \int \int \int_V \text{div} \vec{F} \, dV$

2. Attempt all parts:-

2.a.
$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$
 (CO1,K2) 2

Find the inverse by using elementary transformation where

2.b. Find the first order partial derivatives of $u = \log(x^2 + y^2)$ (CO2, K2) 2

2.c. If $u = x - y, v = 2y + 3z, w = 5z + 2x$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. (CO3,k2) 2

2.d. Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\cot \theta} \, d\theta = \frac{\pi}{\sqrt{2}}$. (CO4,K3) 2

2.e. Find the unit vector normal to the surface $xy^3z^2 = 4$ at the point $(-1, -1, 2)$. (CO5 K2) 2

SECTION-B

3. Attempt all parts:-

3.a. Answer any one of the following:-

3.a.(i) Determine the value of λ and μ such that the system $2x - 5y + 2z = 8, 2x + 4y + 6z = 5, x + 2y + \lambda z = \mu$ has (i) no solution (ii) a unique solution (iii) infinite number of solutions. (CO1,K2) 6

3.a.(ii) Find the rank of matrix $\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$ by reducing it to Normal form. (CO1,K3) 6

3.b. Answer any one of the following:-

3.b.(i) If $u = \log(\tan x + \tan y + \tan z)$, prove that $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$. (CO2,K3) 6

3.b.(ii) If $u = \sec^{-1} \left(\frac{x^3 - y^3}{x + y} \right)$ then evaluate $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$. (CO2,K3) 6

3.c. Answer any one of the following:-

3.c.(i) A balloon is in the form of a right circular cylinder of radius 1.5 m and length 4m, which is surmounted by hemispherical ends. If the radius is increased by 0.01 m and the length by 0.05 m. Find the percentage change in the volume of balloon. (CO3, K3) 6

3.c.(ii) Expand $e^{x \cos y}$ in powers of x and y up to third degree term. (CO3, K2) 6

3.d. Answer any one of the following:-

3.d.(i) Change into polar co-ordinates and hence evaluate the integral 6

$$\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} dy dx \quad . \quad (\text{CO4,K2})$$

3.d.(ii)

Using Beta and Gamma functions, evaluate $\int_0^1 \left(\frac{x^3}{1-x^3} \right)^{1/2} dx.$ (CO4,K3) 6

3.e. Answer any one of the following:-

3.e.(i) Show that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. 6

Find the velocity potential ϕ such that $\vec{A} = \nabla \phi.$ (CO5, K3)

3.e.(ii) Find the directional derivative of $f = 2x^2 + 3y^3 + z^2$ at the point $P(3,1,2)$ in the direction of the vector $\vec{a} = \hat{i} - 2\hat{k}.$ (CO5, K3) 6

SECTION-C 50

4. Answer any one of the following:-

4-a.

Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}.$ Hence compute $A^{-1}.$ 10
 Also find the matrix represented by $A^8 - 11A^7 - 4A^6 + A^5 + A^4 - 11A^3 - 3A^2 + 2A + I.$ (CO1,K2)

4-b.

Find the eigen values and corresponding eigenvectors of a matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ 10
 . (CO1,K3)

5. Answer any one of the following:-

5-a.

If $u = f(r, s, t)$ and $r = \frac{x}{y}, s = \frac{y}{z}, t = \frac{z}{x},$ then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0.$ 10
 (CO2,K2)

5-b.

If $y = (\sin^{-1}x)^2$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0.$ Also find the value of y_n at $x = 0.$ (CO2, K3) 10

6. Answer any one of the following:-

6-a.

If u, v, w are the roots of the cubic equation $(x-a)^3 + (x-b)^3 + (x-c)^3 = 0$ in $x,$ 10
 then find $\frac{\partial(u, v, w)}{\partial(a, b, c)}.$ (CO3, K3)

6-b.

Use the method of Lagrange's multiplier to find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$ (CO3, K3) 10

7. Answer any one of the following:-

7-a.

Evaluate by changing the order of integration $\int_0^a \int_{x^2/a}^{2a-x} xy dy dx$ 10
 . (CO4,K2)

- 7-b. Apply Dirichlet's integral to find the volume and mass contained in the solid region in the first octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, if the density at any point is $\rho(x,y,z) = kxyz$ 10

(CO4,K3)

8. Answer any one of the following:-

- 8-a. Verify Green's theorem in the plane for $\int_C 2y^2 dx + 3x dy$ where C is the boundary of the closed region bounded by $y = x$ and $y = x^2$. (CO5, K3) 10
- 8-b. Verify divergence theorem for $\vec{F} = (x^3 - yz)\hat{i} - 2x^2y\hat{j} + 2z\hat{k}$ taken over the cube bounded by the planes $x = 0, x = a, y = 0, y = a, z = 0, z = a$. (CO5, K3) 10

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