

--	--	--	--	--	--	--	--	--	--	--	--	--	--	--

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M.Tech(Integrated)

SEM: III - THEORY EXAMINATION (2024 - 2025)

Subject: Discrete Structures

Time: 3 Hours

Max. Marks: 100

General Instructions:

IMP: Verify that you have received the question paper with the correct course, code, branch etc.

1. This Question paper comprises of three Sections -A, B, &amp; C. It consists of Multiple Choice Questions (MCQ's) &amp; Subjective type questions.

2. Maximum marks for each question are indicated on right -hand side of each question.

3. Illustrate your answers with neat sketches wherever necessary.

4. Assume suitable data if necessary.

5. Preferably, write the answers in sequential order.

6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

**SECTION-A**

20

1. Attempt all parts:-

- 1-a. For two sets C and D the set  $(C - D) \cap D$  will be \_\_\_\_\_. (CO1, K2) 1
- (a) C  
(b) D  
(c)  $\Phi$   
(d) None of the mentioned
- 1-b. Two sets are called disjoint if there \_\_\_\_\_ is the empty set. (CO1, K1) 1
- (a) Union  
(b) Difference  
(c) Intersection  
(d) Complement
- 1-c. A cyclic group is always an (CO2, K2) 1
- (a) Ring  
(b) Field  
(c) Abelian Group  
(d) Zero Ring
- 1-d. Two groups G and H are isomorphic if: (CO2, K2) 1
- (a) They have the same elements.  
(b) There exists a bijection between G and H that preserves the group operation.

- (c) They are Abelian.
- (d) They are cyclic.
- 1-e. In a Hasse diagram, edges are drawn between elements only if: (CO3, K2) 1
- (a) The elements are equal
- (b) One is the immediate predecessor of the other
- (c) They are unrelated
- (d) The elements are both minimal
- 1-f. A lattice is a poset where every pair of elements has: (CO3, K1) 1
- (a) A unique least upper bound and greatest lower bound
- (b) Only a greatest lower bound
- (c) Only a least upper bound
- (d) Neither upper nor lower bounds
- 1-g. A compound proposition that is always \_\_\_\_\_ is called tautology. (CO4, K1) 1
- (a) TRUE
- (b) FALSE
- (c) both
- (d) none of these
- 1-h. Let P: I am in Delhi.; Q: Delhi is clean.; then  $q \wedge p(q \text{ and } p)$  is: (CO4, K2) 1
- (a) Delhi is clean and I am in Delhi
- (b) Delhi is not clean or I am in Delhi
- (c) I am in Delhi and Delhi is not clean
- (d) Delhi is clean but I am in Mumbai
- 1-i. The chromatic number of a graph is the minimum number of colors required to: (CO5, K2) 1
- (a) Color adjacent edges
- (b) Color adjacent vertices
- (c) Color all vertices
- (d) Color all edges
- 1-j. A graph is planar if: (CO5, K2) 1
- (a) It can be drawn on a plane without edges crossing
- (b) It contains no cycles
- (c) It has an even number of edges
- (d) It contains only vertices with even degree

2. Attempt all parts:-

- 2.a. Define domain and range of a relation. (CO1, K2) 2
- 2.b. Provide an example of a group that is not cyclic. (CO2, K2) 2
- 2.c. Let  $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$  be ordered by relation "a divides b". Draw the Hasse diagram for it. (CO3, K3) 2

2.d. Let  $P(x)$  be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English. (CO4, K3) 2

a)  $\exists xP(x)$

b)  $\forall xP(x)$

2.e. Enlist and brief the conditions under which a graph is planar. (CO5, K3) 2

## **SECTION-B**

30

3. Answer any five of the following:-

3-a. Brief the composition of relations. If two relations  $R_1=\{(1,2),(2,3)\}$  and  $R_2=\{(2,4),(3,5)\}$ , find  $R_1 \circ R_2$  (composition of relations). (CO1, K3) 6

3-b. For  $U=\{1,2,3,4,5,6,7,8,9,10\}$ , let  $A=\{2,4,6,8,10\}$  and  $B=\{1,2,3,4,5\}$ . Find:  
a)  $A \Delta B$  (symmetric difference),  
b) complement of  $(A \cup B)$  6

3-c. Given  $G=\{1,2,3,4\}$  under multiplication modulo 5, find the inverse of each element in G. 6

3-d. Show that a subgroup of a cyclic group is cyclic. 6

3.e. Compare and contrast bounded and unbounded lattices with examples. 6

3.f. Write rules of inference for i) Modus tollens ii) Disjunctive syllogism (CO4) 6

3.g. Explain in depth the different methods of representing a graph (adjacency matrix, adjacency list, incidence matrix) and the advantages/disadvantages of each. 6

## **SECTION-C**

50

4. Answer any one of the following:-

4-a. Define a relation and explain reflexive, symmetric, and transitive properties. Verify these properties for  $R=\{(1,1),(2,2),(3,3),(1,2),(2,1)\}$  on  $A=\{1,2,3\}$ . (CO1, K3) 10

4-b. Let  $R=\{(a,b) | a+b=10\}$  on  $A=\{1,2,3,4,5,6,7,8,9\}$ . Find all pairs in R and check if R is symmetric or transitive. (CO1, K3) 10

5. Answer any one of the following:-

5-a. Discuss a cyclic group. Prove that every cyclic group is Abelian Group and also provide an example for this. (CO2, K3) 10

5-b. Define group homomorphism. Show that the identity element is always mapped to the identity element in the codomain. (CO2, K3) 10

6. Answer any one of the following:-

6-a. Explain the way distributive lattices and complemented lattices are used in Boolean algebra. Sketch a lattice that is distributed and complemented both. (CO3, K4) 10

6-b. Brief the POSET of  $D_{42}$  and also draw the Hasse diagram for the POSET. (CO3, K4) 10

7. Answer any one of the following:-

- 7-a. Let  $F(x, y)$  be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements. (CO4, K3) 10
- a) Everybody can fool Fred.
  - b) Evelyn can fool everybody.
  - c) Everybody can fool somebody.
  - d) There is no one who can fool everybody.
- 7-b. Let  $p$  and  $q$  be the propositions as: (CO4, K3) 10
- $p$ : You drive over 65 miles per hour.  
 $q$ : You get a speeding ticket.
- Write these propositions using  $p$  and  $q$  and logical connectives (including negations).
- a) You do not drive over 65 miles per hour.
  - b) You drive over 65 miles per hour, but you do not get a speeding ticket.
  - c) You will get a speeding ticket if you drive over 65 miles per hour.
  - d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.
  - e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.
8. Answer any one of the following:-
- 8-a. Differentiate walk and path. Discuss Euler's theorem and also evaluate the significance of Euler's Theorem in finding Eulerian Circuits with a real-world example. (CO5, K4) 10
- 8-b. Define Hamiltonian paths and circuits. Compare and contrast the conditions for the existence of a Hamiltonian path and a Hamiltonian circuit. (CO5, K4) 10