Printed Page:-04 **Subject Code:- BMIAS0301A**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M.Tech(Integrated)

SEM: III THEORY EXAMINATION (2024- 2025)

Subject: Engineering Mathematics-III

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* ***three Sections -A, B, & C.*** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION-A 20

1. Attempt all parts:-

1-a. The solution of one dimension wave equation is: (CO1, K2) 1

(a) $y(x,t) = (c_1 \cos px + c_2 \sin px)(c_3 \cos cpt + c_4 \sin cpt)$

(b) $y(x,t) = (c_1\, e^{px} + c_2 e^{-px})(c_3\, e^{cpt} + c_4 e^{-cpt})$

(c) $y(x,t) = (c_1 + c_2\, x)(c_3\, t + c_4)$

(d) None of these

1-b. The order of Partial Differential equation $\frac{\partial^2 u}{\partial t^2} + 2\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 u}{\partial x^2} = 0$ is: (CO1, K2) 1

(a) 1

(b) 2

(c) 3

(d) 4

1-c. A multiple of 3 number of sub intervals is required for _____. (CO2, K1) 1

(a) Simpson's 1/3rd rule

(b) Simpson's 3/8th rule

(c) Trapezoidal rule

(d) All of these

.

1-d. Gauss Seidel method is applicable to _____ matrix. (CO2, K1) 1

(a) Null Matrix
(b) Identity Matrix
(c) Triangular Matrix
(d) Diagonally Dominant Matrix

1-e. u(x.y) is given then find v(x,y) by the following equation (CO3, K1) 1

(a) $dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy$

(b) $dv = \frac{\partial v}{\partial x} dx + i\frac{\partial v}{\partial y} dy$

(c) $dv = \frac{\partial v}{\partial x} dx + \frac{\partial u}{\partial y} dy$

(d) None of these

1-f. $f(z) = u + iv$ is an analytic function of z then u and v are (CO3, K1) 1

(a) Not Harmonic
(b) May be Harmonic
(c) Harmonic function
(d) None of these

1-g. In Laurent's expansion $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n + \sum_{n=1}^{\infty} b_n (z-a)^{-n}$, the second summation is called (CO4, K1) 1

(a) Null part
(b) Principal Part
(c) Analytic Part
(d) None of these

1-h. *The value of* $\frac{1}{2\pi i}\int_C \frac{e^z}{(z-2)^3} dz$, C: $|z| = 5/2$, is (CO4, K3) 1

(a) $\frac{1}{2}e^2$
(b) $2\pi i\, e^2$
(c) e^2
(d) e

1-i. Words can be formed by using all letters of the word "BIHAR" is…. (CO5, K3) 1

(a) 210
(b) 120
(c) 230
(d) None of these

1-j. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, P = {1, 2, 5}, Q = {6, 7}. Then P ∩ Q' is : (CO5, K3) 1

.

(a) P

(b) Q

(c) P'

(d) Q'

2. Attempt all parts:-

2.a. Classify the PDE $u_{xx} + 3u_{xy} + u_{yy} = 0$. (CO1, K2) 2

2.b. Evaluate $\int_0^3 f(x)dx$ using trapezoidal rule, for the following table: (CO2, K3) 2

x	0	1	2	3
f(x)	1	0.5	0.2	0.1

2.c. Write formula for derivative of f(z) at origin (CO3, K1) 2

2.d. Write Cauchy's integral formula for complex integration.(CO4, K1) 2

2.e. How many words can be formed from the letters of the word "DIRECTOR" So that the vowels are always together? (CO5, K3) 2

SECTION-B 30

3. Answer any five of the following:-

3-a. Solve: $2r - s - 3t = 5\frac{e^x}{e^y}$.(CO1, K3) 6

3-b. Solve the following PDE by method of separation of variables: 6

$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$; $u(0,y) = 8e^{-3y}$.(CO, K3)

3-c. Solve the following system of equations by Gauss elimination method: (CO2, K3) 6

$$x_1 - 2x_2 + 9x_3 = 8$$

$$2x_1 - 8x_2 + x_3 = -5$$

$$3x_1 + x_2 - x_3 = 3$$

3-d. Evaluate $\int_0^6 \frac{dx}{(1+x^2)}$ by using (CO2, K3) 6

i) Trapezoidal rule ii) Simpson's 3/8 rule.

3.e. Show that the function $u(x,y) = e^x \sin y$ is harmonic. Find its harmonic conjugate.(CO3, K2) 6

3.f. Evaluate: $\oint_{1-i}^{2+3i} (z^2 + z)\,dz$, along the line joining the points $(1,-1)$ and $(2,3)$.(CO4, K3) 6

3.g. For a positive integer n, let P_n denote the product of the digits of n and S_n denote the sum of the digits of n. find the number of integers between 10 and 1000 for which $P_n + S_n = n$. (CO5, K3) 6

SECTION-C 50

4. Answer any one of the following:-

4-a. Solve: $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$. (CO1, K3) 10

4-b. A tightly stretched string with fixed end points *x=0* and *x=l* is initially in a position given by $y = y_0 \sin^3 \frac{\pi x}{l}$. If it is released from rest from this position, find the displacement $y(x,t)$. (CO1, K3) 10

5. Answer any <u>one</u> of the following:-

5-a. Find a square root of 12 by Newton-Raphson method, correct to 4 decimal places. (CO2, K3) 10

5-b. Using Lagrange interpolation formula find *f*(10) from the following table: (CO2, K3) 10

x	5	6	9	11
f	12	13	14	16

6. Answer any <u>one</u> of the following:-

6-a. Examine the nature of the function $f(z) = \frac{xy^2(x+iy)}{(x^2+y^4)}$ $z \neq 0$ and $f(z) = 0$ in the region including the origin. (CO3, K3) 10

6-b. If $f(z)$ is an analytic function then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \text{Re } f(z)^2 = 2f'(z)^2$. (CO3, K2) 10

7. Answer any <u>one</u> of the following:-

7-a. Verify Cauchy's integral theorem for the function $f(z) = z^2$ along the boundary of the square with the vertices at the points *1+i, -1+i, 1-i* and *-1-i*. (CO4, K2) 10

7-b. Evaluate by using Cauchy's Residue theorem : $\int_C \frac{z^2}{(z-1)^2(z+2)} dz$ where C is the circle $|z| = 3$. (CO4, K3) 10

8. Answer any <u>one</u> of the following:-

8-a. Find the maximum power of 3 in the expansion of 1! × 2! × 3! × . . . . × 100!. (CO5, K3) 10

8-b. Answer the questions based on the following table. (CO5, K3) 10

The table gives the production of major agricultural products in Million Tonnes (MT).

1. The agro product which witnessed the highest growth rate in production from 1997 to 2000 is ____
2. Pulses production in 1998 is what percent of the total production of rice in the given 4-year period?
3. By what percent is the average wheat production more than the average sugar cane production for the given 4-year period?
4. What is the simple annual growth rate of wheat from 1997-2000 ?
5. What would be the actual production of wheat in 2001 if the growth in 2001 is the same as the average growth for the period?

Year	Wheat	Rice	Sugar cane	Pulses
1997	100	91	15	71
1998	120	88	18	75
1999	125	97	21	79
2000	131	107	25	88

.