Printed Page:-05 **Subject Code:- BMIAS0103**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

M.Tech(Integrated)

SEM: I - THEORY EXAMINATION (2024- 2025)

Subject: Engineering Mathematics-I

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* ***three Sections -A, B, & C.*** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION-A 20

1. Attempt all parts:-

1-a. 1

The largest eigen value of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 5 \\ 0 & 0 & 4 \end{bmatrix}$ is (CO1,K1)

(a) 1

(b) 5

(c) 4

(d) None of these

1-b. If A is matrix such that there exists a square submatrix of order r which is non-singular and every submatrix of order r+1 or more is singular, then (CO1,K2) 1

(a) rank A = r+1

(b) rank A = r

(c) rank A > r

(d) $rank\ A \geq r+1$

1-c. 1

The degree of homogeneous function $u(x,y) = x^{\frac{1}{3}} y^{\frac{-4}{3}} \sin^{-1}(\frac{y}{x})$ is (CO2,K2)

(a) -1

(b) 1

(c) 4

.

(d) -4/9

1-d. If $y = \sin(m\sin^{-1}x)$ then after two time differentiation we get (CO2,K3) 1

(a) $(1-x^2)y_2 - xy_1 - m^2 y = 0$

(b) $(1-x^2)y_2 - xy_1 + m^2 y = 0$

(c) $(1-x^2)y_2 + xy_1 + m^2 y = 0$

(d) None of these.

1-e. The Taylor's series of f(x) at point 'a' is (CO3,K2) 1

(a) $\mathbf{f(a) + \frac{(x-a)}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \cdots}$

(b) $\mathbf{f(a) + \frac{x}{1!}f'(a) + \frac{x^2}{2!}f''(a) + \frac{x^3}{3!}f'''(a) + \cdots}$

(c) $\mathbf{f(0) + \frac{(x-a)}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(0) + \frac{(x-a)^3}{3!}f'''(0) + \cdots}$

(d) None of these

1-f. The period T of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$, the error in L is 2% and g is constant. The corresponding error in T is (CO3,K2) 1

(a) 2

(b) 1

(c) 15

(d) 18

1-g. If u= x-y and v= x+y, then the area element dxdy is replaced by (CO4,K2) 1

(a) $\frac{1}{2}dudv$

(b) $\frac{1}{3}dudv$

(c) $dudv$

(d) None of these

1-h. By changing order of integration $I = \int_0^1 \int_{e^x}^{e} f(x,y)\,dydx$ leads to the value $I = \int_r^s \int_p^q f(x,y)\,dxdy$. The value of q is (CO4,K3) 1

(a) 0

(b) log y

(c) y

(d) None of these

1-i. A's salary is 20% more than B's ; B's salary is 10% less than C's .If A's salary is Rs.1080 , find C's salary (CO5,K3)) 1

(a) 900

.

(b) 1000

(c) 1200

(d) None of these

1-j. If "PROMPT" is coded as QSPLOS , then " PLAYER" should be (CO5,K2) 1

(a) QMBZFS

(b) QMBXDQ

(c) QUREXM

(d) URESTI

2. Attempt all parts:-

2.a. Show that the matrix $\begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$ is Hermitian matrix. (CO1,K2) 2

2.b. Find the nth derivative of $y = \frac{1}{(2x+3)(3x-1)}$ (CO2,K1) 2

2.c. Find the stationary points of $f(x,y) = 5x^2 + 10y^2 + 12xy - 4x - 6y + 1$. (CO3,K2) 2

2.d. Change into polar coordinates $\int_0^a \int_0^{\sqrt{a^2-x^2}} f(x,y)\,dy\,dx$. (CO4,K1) 2

2.e. The average of 12 numbers is 9. If each number is multiplied by 2 and added to 3. Find the average of the new set of numbers? (CO5,K2) 2

SECTION-B 30

3. Answer any five of the following:-

3-a. Show that the system of equations $3x+4y+5z=a,\ 4x+5y+6z=b,\ 5x+6y+7z=c$ does not have a solution unless a + c = 2b. (CO1,K2) 6

3-b. Find the characteristic roots of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and verify Caley-Hamilton theorem for this matrix. Find A^{-1} also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A. (CO1,K3) 6

3-c. If $u = \sin^{-1}\left(\frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -3\tan u$ (CO2,K2) 6

3-d. If $u = x^2\tan^{-1}\left(\frac{x}{y}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$, $x,y \neq 0$. Then show that $u_{xy} = u_{yx}$. (CO2,K3) 6

3.e. If $u^3+v^3+w^3 = x+y+z,\ u^2+v^2+w^2 = x^3+y^3+z^3$ and $u+v+w = x^2+y^2+z^2$ then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$. (CO3,K3) 6

3.f. Change into polar co-ordinates and hence evaluate the integral 6

$\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dy\,dx$. (CO4,K2)

3.g. The marked price of a pencil is 35% more than its cost price. What maximum discount percentage can be offered by the shopkeeper to sell his pencil at no profit or no loss? (CO5,K1) 6

SECTION-C 50

4. Answer any one of the following:-

4-a. Show that the matrix $A=\begin{bmatrix}1 & -2 & 1\\ 1 & -2 & 3\\ 0 & -1 & 2\end{bmatrix}$ satisfies its own characteristic equation and hence find A^{-1}. (CO1,K2) 10

4-b. Find the rank of the matrix $\begin{bmatrix}2 & 1 & 3 & 4\\ 0 & 3 & 4 & 1\\ 2 & 3 & 7 & 5\\ 2 & 5 & 11 & 6\end{bmatrix}$ by reducing it to normal form. (CO1,K1) 10

5. Answer any one of the following:-

5-a. If $V=\log_e \sin\left\{\frac{\pi(2x^2+y^2+xz)^{1/2}}{2(x^2+xy+2yz+z^2)^{1/3}}\right\}$, prove that when $x=0, y=1, z=2$

$$\left(x\frac{\partial V}{\partial x}+y\frac{\partial V}{\partial y}+z\frac{\partial V}{\partial z}\right)=\frac{\pi}{12}$$

(CO2,K3) 10

5-b. If $u = f(r)$, where $r=\sqrt{x^2+y^2}$, prove that

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = f''(r)+\frac{1}{r}f'(r).$$ (CO2,K2) 10

6. Answer any one of the following:-

6-a. Use the method of Lagrange's multiplier to find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid whose equation is $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. (CO3,K2) 10

6-b. In estimating the number of bricks in a pile which is measured to be (5 m x 10 m x 5 m) the count of bricks is taken as 100 bricks/meter^3. Find the error in the cost when the tape is stretched 2% beyond its standard length. The cost of bricks is Rs. 2000 per thousand bricks. (CO3,K1) 10

7. Answer any one of the following:-

7-a. Evaluate $\int\int_R (x+y)^2 dxdy$ where R is the parallelogram in xy plane with vertices (1, 0), (3, 1), (2, 2) and (0, 1) using the transformation u = x+y and v = x - 2y. (CO4,K2) 10

7-b. Evaluate $\int\int\int x^2yz\ dx\ dy\ dz$, throughout the volume bounded by the planes $x = 0, y = 0, z = 0\ and\ \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. (CO4,K2) 10

8. Answer any <u>one</u> of the following:-

8-a. (a) If in certain code 1326 is coded as 8673, and 5670 is coded as 4329 then find the code for 0009 ? 10
(b) The total population of a village is 5000. The number of male and female increases by 10% and 15% respectively and consequently the population of the village become 5600. What was the number of males in the village?
(c) A dealer offers a discount of 10% on the marked price of an article and still makes a profit of 20%. If its marked price is Rs. 800, then find the cost price ? (CO5,K1)

8-b. (a) The average of 6 persons in a committee is increased by 2 years, when two men aged 55 years and 60 years are substituted by two women. Find the average age of these two women .? 10
(b) In certain code language SERIES is coded as 5625 and PIPE is coded as 2116. How will WAP be coded in the same code language?
(c) If a watch is sold at Rs.60, there is a loss of 15% for a profit of 2%, the watch is to be sold at what ? (CO5,K2)