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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: II - THEORY EXAMINATION (2024 - 2025)

Subject: Mathematical Foundations – II

Time: 3 Hours

Max. Marks: 100

General Instructions:

IMP: Verify that you have received the question paper with the correct course, code, branch etc.

1. This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.

2. Maximum marks for each question are indicated on right -hand side of each question.

3. Illustrate your answers with neat sketches wherever necessary.

4. Assume suitable data if necessary.

5. Preferably, write the answers in sequential order.

6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION-A

20

1. Attempt all parts:-

1-a. The gamma function of n ($n > 0$) is denoted by $\Gamma(n)$ and defined as (CO1, K2) 1

(a) $\int_0^{\infty} x^n e^{-x} dx$

(b) $\int_{-\infty}^{\infty} x^n e^{-x} dx$

(c) $\int_0^{\infty} x^{n-1} e^{-x} dx$

(d) None of these

1-b. The value of the triple integral $\int_0^1 \int_1^2 \int_2^3 xyz \, dx \, dy \, dz$ is (CO1, K2) 1

(a) $12/7$

(b) $15/8$

(c) 1

(d) 0

1-c. The complementary function of the second order linear differential equation $x^2 y'' - xy' = 0$ is : (CO2, K3) 1

(a) $c_1 + xc_2$

(b) $c_1 + x^2 c_2$

(c) $c_1 + e^x c_2$

(d) None of these

1-d. The P.I. of the diff. equation $(D^2 + 4)y = \sin 3x$ is : (CO2, K2)

1

(a) $-\frac{1}{10} \sin 3x$

(b) $-\frac{1}{5} \sin 3x$

(c) $\frac{1}{5} \sin 3x$

(d) None of these

1-e. The Complementary function of partial differential equation

1

$(D - D' - 1)(D - D' - 3)z = 0$ is (CO3, K3)

(a) $C.F. = e^x f_1(y+x) + e^{-3x} f_2(y+x)$

(b) $C.F. = e^x f_1(y+x) + e^{-3x} f_2(y-x)$

(c) $C.F. = e^x f_1(y+x) + e^{3x} f_2(y+x)$

(d) $C.F. = e^x f_1(y-x) + e^{3x} f_2(y+x)$

1-f. Which of the following is the correct partial differential equation of the relation

1

$z = (x-a)^2 + (y-b)^2$, where 'a' and 'b' are constants. (CO3, K2)

(a) $pq + p + q = z$

(b) $p + q + ab = 2z$

(c) $px + qy + pq = 4z$

(d) $p^2 + q^2 = 4z$

1-g. Inverse Laplace of the function $f(s) = \frac{e^{-s}}{s}$ is (CO4, K3)

1

(a) $u(t-1)$

(b) $u(t+1)$

(c) $-u(t+1)$

(d) None of these

1-h. Laplace transform of te^{-t} is (CO4, K3)

1

(a) $\frac{1}{(s+1)^2}$

(b) $\frac{2}{(s+1)^2}$

(c) $\frac{1}{(s-1)^2}$

(d) None of these

- 1-i. Sandhya starting from her house, goes 4 km in the East, then she turns to her right and goes 3 km. What is the shortest distance to reach her house? (CO5,K2) 1
- (a) 1 Km
(b) 7 Km
(c) 5 Km
(d) None of these
- 1-j. If $a:b = 3:4$ and $b:c = 4:7$ then $a:c =$ (CO5,K2) 1
- (a) 7:3
(b) 4:7
(c) 3:7
(d) 7:11

2. Attempt all parts:-

- 2.a. Evaluate the value of $\int_0^1 x^4(1-x)^3 dx$. (CO1, K2) 2
- 2.b. Find the particular integral of differential equation $(D^2 - 1)y = \cos x$. (CO1,K2) 2
- 2.c. Solve the partial differential equation $(D^2 - DD' - 6D'^2)z = 0$. (CO3, K3) 2
- 2.d. Find the inverse Laplace transform of the function $f(s) = \frac{1}{\sqrt{s+4}} + \frac{1}{s}$. (CO4, K3) 2
- 2.e. The prices of a scooter and a television set are in the ratio 3:2. If a scooter costs Rs.6000 more than the television set, then find the price of the television set? (CO5, K2) 2

SECTION-B

3. Answer any five of the following:-

- 3-a. Evaluate $\iint_R xy \, dx \, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$. (CO 1, K3) 6
- 3-b. Evaluate the integral $\int_0^1 \int_{y^2}^y (1 + xy^2) \, dx \, dy$. (CO 1, K3) 6
- 3-c. Solve: $(D^2 - 3D + 2)y = x^2 + 2x + 1$. (CO2, K3) 6
- 3-d. Solve the differential equation: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \sin x$. (CO2,K3) 6
- 3.e. Solve the linear partial differential equation $(D - 3D' - 2)^2 z = 2e^{2x} \sin(3x + y)$. (CO3, K3) 6
- 3.f. Find inverse Laplace Transform of the function $f(s) = \log \frac{s(s+1)}{s^2+4}$. (CO 4, K3) 6
- 3.g. A man invested Rs. 16000 at compound interest for 3 years, interest compounded annually. If he got Rs. 18522 at the end of 3 years, what is rate of interest? (CO5, K2) 6

SECTION-C

50

4. Answer any one of the following:-

4-a. Evaluate by changing the order of integration $\int_0^a \int_{x^2/a}^{2a-x} xy \, dy \, dx$. (CO 1, K3) 10

4-b. Evaluate the triple integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dz \, dy \, dx$. (CO1,K3) 10

5. Answer any one of the following:-

5-a. Solve : $\frac{dx}{dt} + 2x - 3y = t$, $\frac{dy}{dt} - 3x + 2y = e^t$. (CO2,K3) 10

5-b. Solve the following differential equation by changing the independent variable:
 $\frac{d^2y}{dx^2} - \frac{1}{x} \frac{dy}{dx} + 4x^2y = x^4$. (CO2,K3) 10

6. Answer any one of the following:-

6-a. Solve : $(D + D' - 1)(D + D' - 3)(D + D')z = e^{x+y} \sin(2x + y)$. (CO3,K3) 10

6-b. Solve the linear partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2 \frac{\partial^2 z}{\partial y^2} = (y - 1)e^x$ (CO3,K3) 10

7. Answer any one of the following:-

7-a. Using Convolution Theorem find the inverse Laplace transform of $L^{-1} \left\{ \frac{s}{(s^2 + 1)(s^2 + 4)} \right\}$. (CO4,K3) 10

7-b. Solve the following differential equation by using Laplace transform $\frac{d^3x}{dt^3} - 3 \frac{d^2x}{dt^2} + 3 \frac{dx}{dt} - x = t^2 e^t$, Given that $x(0) = 1$, $x'(0) = 0$, $x''(0) = -2$. (CO 4, K4) 10

8. Answer any one of the following:-

8-a. (i) There are two containers of equal capacity. The ratio of milk to water in the first container is 3:1, in the second container 5:2. If they are mixed up, find the ratio of milk to water in the mixture. 10

(ii) S.I on a sum for 3yrs at any rate of interest is ₹ 450 while C.I on the same sum at the same rate for 2 yrs is ₹ 315. Find the sum and rate percent. (CO5,K2)

8-b. (i) I was facing East from where I turned to my left and walked 12 feet then I turned towards right and walked 6 feet. After that I walked 6 feet in South direction and at last walked 6 feet in the West. Then, in which direction am I standing from the original point? 10

(ii) The milk and water in a mixture are in the ratio 7:5. When 15 L of water are added to it, the ratio of milk and water in the new mixture becomes 7:8. Find the total quantity of water in the new mixture. (CO5,K2)