Printed Page:-05

Subject Code:- BAS0104

202

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: I - THEORY EXAMINATION (2024- 2025)

Subject: Mathematical Foundations-I

Time: 3 Hours General Instructions:

IMP: Verify that you have received the question paper with the correct course, code, branch etc.
1. This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.
2. Maximum marks for each question are indicated on right -hand side of each question.
3. Illustrate your answers with neat sketches wherever necessary.

4. Assume suitable data if necessary.

5. Preferably, write the answers in sequential order.

6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION-A

1. Attempt all parts:-

- 1-e. The nth derivative of $\log(ax+b)$ is (CO3,K1)
 - $\frac{(-1)^n n! a^n}{(ax+b)^n}$

(a)
$$(-1)^{n-1} n! a$$

b)
$$(ax + b)^{n+1}$$

 $(-1)^n (n-1)! c$

(c)
$$(ax+b)^n$$

(d) $(ax+b)^n$
 $(ax+b)^n$

1-a. If the eigen values of a matrix A are 4, 5, 7 then write the eigen values of A^{-1} are 1 (CO1,K1)

- (a) 4, 5^2 , 7^3
- (b) 4, 5, 7
- (c) 1/4, 1/5, 1/7
- (d) none of these

1-f. If
$$u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$$
, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is (CO3,K2)

- (a) 0
- (b) u
- (c) 4
- (d) 1

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Max. Marks: 100

20

1

1

If $AA^{\theta} = I = A^{\theta}A$, then A is (CO1,K1)) 1-b. Hermitian matrix (a) (b) skew-Hermitian Unitary (c) none of these (d) If $x = r\cos\theta$ and $y = r\sin\theta$ then the value of jacobian $\frac{\partial(x,y)}{\partial(r,\theta)}$ is (CO4,K2) 1-g. 1 (a) r (b) 2r 1 (c) r (d) 1 In an inner product space, which option is correct ? (CO2,K1) 1-c. $(\alpha, a\beta + b\gamma) = a(\alpha, \beta) + b(\beta, \gamma)$ (a) $(\alpha, a\beta + b\gamma) = (a, a\beta) + (a, b\gamma)$ (b) $(\alpha, a\beta + b\gamma) = a(\alpha, \beta) + b(\alpha, \gamma)$ (c) (d) None of these An error of 2% is made in measuring length and breadth then the percentage error 1-h. in the area of the rectangle is (CO4,K3) 202 (a) 6 4 (b) (c) 8 (d) 16 Every superset of a linearly dependent set of vectors is 1 1-d. (CO2,K1) Dependent (a) (b) Independent (c) Both None of these (d) Which term of the series $1+2+4+8+\dots$ is 256 (CO5,K2) 1-i. 1 (a) 9 (b) 8 (c) 10

1

1

1

- (d) 7
- 1-j.In an examination, 50% students failed in English and 40% in math and 15%1students failed in both the subjects. If 200 students passed in both the subjects,

find the number of students appeared in the examination? (CO5,K2)

- (a) 500
- (b) 600
- (c) 800
- (d) 1000

2. Attempt all parts:-

2.a.	$A = \begin{bmatrix} 2 \end{bmatrix}$	3 – 4i	2
	Show that the matrix $\begin{bmatrix} A & - \\ 3 + 4i \end{bmatrix}$ Hermitian matrix. (CO1,K2)	2 \int is Hermitian matrix and <i>iA</i> is a skew-	
2 h	State the rank and nullity theorem	(CO2 K1)	2

2.b.	State the rank and nullity theorem.	. (CO2,K1)	2

2.c.	=	
	Find the nth derivative of v^{-}	(2x+3)(3x-1) (CO3,K3)

2.d. Expand e^x in powers of (x-2). (CO4,K3)

2.e. A man earns Rs.4000 in a month and saves 30% of his income. What is his

expenditures in a year? (CO5,K2)

SECTION-B

30

6

6

50

2

2

2

3. Answer any five of the following:-

3-a. Find the inverse of the matrix by using elementary transformations, where $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$

356 . (CO1.K2)

2	6	-2	4	6	
rix 0	11	-5	3		(CO1 V2)
1 - 5 3 1 by reducing it to Echelon form	6 (CO1,K2)				
_4	1	1	5		
t	trix 0 1 4	trix $\begin{bmatrix} 2 & 6 \\ 0 & 11 \\ 1 & -5 \\ 4 & 1 \end{bmatrix}$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	trix $\begin{bmatrix} 2 & 6 & -2 & 4 \\ 0 & 11 & -5 & 3 \\ 1 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$ by reducing it to Echelon form.

- 3-c. Prove that two vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ in a real inner product space are orthogonal if and 6 only if $\|\boldsymbol{\alpha} + \boldsymbol{\beta}\|^2 = \|\boldsymbol{\alpha}\|^2 + \|\boldsymbol{\beta}\|^2$. (CO2,k2)
- 3-d. Show that the mapping $T: (a, b) \rightarrow (a+2, b+3)$ of $V_2(R)$ into itself is not a linear transformation. (CO2,K2)

3.e. If
$$\mathbf{x}^{\mathbf{x}} \mathbf{y}^{\mathbf{y}} \mathbf{z}^{\mathbf{z}} = \mathbf{c}$$
 then Show that at $\mathbf{x} = \mathbf{y} = \mathbf{z}$,
 $\left(\frac{\partial^2 \mathbf{z}}{\partial \mathbf{x} \partial \mathbf{y}}\right) = -(\mathbf{x} \log \mathbf{x})^{-1}$ (CO3,K2)

3.f.

Expand $e^{x \sin y}$ in the powers of x and y in the neighborhood of $\begin{pmatrix} 0, \frac{\pi}{4} \end{pmatrix}$ up to the third degree terms. (CO4,K2)

3.g. Average age of 9 members of a club is 29 years. If 2 more persons with the 6 average age of 40 years have become the members of the club, find average age of all the 11 members? (CO5,K2)

SECTION-C

4. Answer any <u>one</u> of the following:-4-a. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence 10

compute A^{-1} . Also evaluate $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$. (CO1,K3).

4-b. Investigate for what values of λ and μ the equations x + 2y + z = 8, 2x + 2y + 2z = 13 and $3x + 4y + \lambda z = \mu$ have (i) no solution (ii) unique solution (iii) many solutions. (CO1,K3)

10

10

- 5. Answer any one of the following:-
- 5-a. Find the basis and dimension of the subspace W of \mathbb{R}^4 generated by (1, 4, -1, 3), (2, 1, -3, -1) and (0, 2, 1, -5). (CO2,K3)
- 5-b. Find the range, rank and nullity of the linear transformation $T = R^3 \rightarrow R^2$ defined 10 by $T(x_1, x_2, x_3) = (x_1 + x_2, 2x_3 - x_1)$ (CO2,K3)

6. Answer any one of the following:-

6-a.
If
$$\mathbf{u} = \mathbf{f}(\mathbf{r}, \mathbf{s}, \mathbf{t})$$
 and $\mathbf{r} = \frac{\mathbf{x}}{\mathbf{y}}, \mathbf{s} = \frac{\mathbf{y}}{\mathbf{z}}, \mathbf{t} = \frac{\mathbf{z}}{\mathbf{x}}$, then show that
 $\mathbf{x} \frac{\partial \mathbf{u}}{\partial \mathbf{x}} + \mathbf{y} \frac{\partial \mathbf{u}}{\partial \mathbf{y}} + \mathbf{z} \frac{\partial \mathbf{u}}{\partial \mathbf{z}} = \mathbf{0}.$ (CO3,K2)

6-b. If $\mathbf{y} = \mathbf{e}^{\mathbf{a} \cos^{-1} \mathbf{x}}$, then prove that $(1 - \mathbf{x}^2) \mathbf{y}_{n+2} - (2n+1) \mathbf{x} \mathbf{y}_{n+1} - (n^2 + \mathbf{a}^2) \mathbf{y}_n = \mathbf{0}$. Calculate $\mathbf{y}_n^{(0)}$ (CO3,K3)

7. Answer any one of the following:-

- 7-a. Find the maximum and minimum distances of the point (3,4,12) 10 from the sphere $x^2 + y^2 + z^2 = 1$ (CO4,K3))
- 7-b. If u, v, w are the roots of the cubic $(\mathbf{x} \mathbf{a})^3 + (\mathbf{x} \mathbf{b})^3 + (\mathbf{x} \mathbf{c})^3 = 0$ then find 10 $\frac{\partial(\mathbf{u}, \mathbf{v}, \mathbf{w})}{\partial(\mathbf{a}, \mathbf{b}, \mathbf{c})} \cdot_{(CO4, K3)}$
- 8. Answer any one of the following:-
- 8-a. (a) After getting two successive discounts Shalini got a shirt at Rs. 136 whose 10 marked price is Rs. 200. If the second discount is 15% find the first discount?
 (b) In a certain code language, COMPUTER is written as KWUXCBMZ, then what should we write for RUNJHUN?

(c) Average marks of group of students is 48. Out of these, 3 students with marks 43, 68 and 51 are removed and a new student with a score of 84 is added to the list. If the number of students in the group was 8, then find the percentage increase in the average marks with respect to the initial average? (CO5,K3)

8-b. (a) The average of 6 persons in a committee is increased by 2 years, when two 10 men aged 55 years and 60 years are substituted by two women. Find the average age of these two women .?

(b) In certain code language SERIES is coded as 5625 and PIPE is coded as 2116. How will WAP be coded in the same code language?

(c) If a watch is sold at Rs.60, there is a loss of 15% for a profit of 2%, the watch is to be sold at what ? (CO5,K3))

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