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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: IV - THEORY EXAMINATION (2024 - 2025)

Subject: Engineering Mathematics-III

Time: 3 Hours

Max. Marks: 100

General Instructions:**IMP:** Verify that you have received the question paper with the correct course, code, branch etc.1. This Question paper comprises of **three Sections -A, B, & C**. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.

2. Maximum marks for each question are indicated on right -hand side of each question.

3. Illustrate your answers with neat sketches wherever necessary.

4. Assume suitable data if necessary.

5. Preferably, write the answers in sequential order.

6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION-A

20

1. Attempt all parts:-

1-a. The function $u = e^{bx} \cos 5y$ is harmonic then the value of b is... (CO1,K3)

1

(a) ± 10 (b) ± 1

(c) 5

(d) ± 5 1-b. If Imaginary part of an analytic function $f(z) = u + iv$ is given then Milne's Thomson formula is: (CO1,K1)

1

(a) $f(z) = \int \psi_1(z,0)dz + \int \psi_2(z,0)dz + c$ (b) $f(z) = \int \psi_1(z,0)dz + i \int \psi_2(z,0)dz + c$ (c) $f(z) = \int \psi_2(z,0)dz - i \int \psi_1(z,0)dz + c$

(d) None of these

1-c. If $f(z) = \frac{\sin z}{z}$, then $z = 0$ is (CO2,K3)

1

(a) Removable singularity

(b) Non-Isolated singularity

(c) Essential singularity

(d) None of these

- 1-d. Let $f(z) = \frac{z^2 + 5z + 6}{z - 2}$ and also let C_1 is $|z| = 3$ and C_2 is $|z| = 1$, then (CO2,K3) 1
- (a) $\int_{C_1} f(z) dz = 0$ and $\int_{C_2} f(z) dz \neq 0$.
- (b) $\int_{C_1} f(z) dz \neq 0$ and $\int_{C_2} f(z) dz \neq 0$.
- (c) $\int_{C_1} f(z) dz \neq 0$ and $\int_{C_2} f(z) dz = 0$.
- (d) $\int_{C_1} f(z) dz = 0$ and $\int_{C_2} f(z) dz = 0$.
- 1-e. Solution of the PDE $(D + D')(D + 2D' + 1)Z = 0$ is (CO3,K3) 1
- (a) $z = f_1(y) + f_2(x) + e^x f_3(y - 2x)$
- (b) $z = f_1(y) + f_2(x) + e^x f_3(y + 2x)$
- (c) $z = f_1(x) + e^{-x} f_2(y - 2x)$
- (d) $z = f_1(y - x) + e^{-x} f_2(y - 2x)$
- 1-f. The following is a heat equation of 2-dimensional in steady state. (CO3,K1) 1
- (a) $\frac{\partial u}{\partial t} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$
- (b) $u = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$
- (c) $\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$
- (d) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
- 1-g. If $f(p)$ is the complex Fourier transform of $F(x)$ then $F[F(x - a)]$ (CO4,K1) 1
- (a) $f(ap)$
- (b) $e^{iap} f(p)$
- (c) $\frac{1}{2} [f(p + a) - f(p - a)]$
- (d) $f(p + a) - f(p - a)$
- 1-h. If the unit step function is defined by $u(k) = \begin{cases} 1, & k \geq 0 \\ 0, & k < 0 \end{cases}$ then $Z\{u(k)\}$ is (CO4,K1) 1
- (a) $\frac{z}{z - 1}$
- (b) $\frac{1}{z}$

- (c) $\frac{1}{z-1}$
 (d) 1

- 1-i. January 1, 2007 was Monday. The week day lies on Jan. 1, 2008 is... (CO5,K2) 1
 (a) Monday
 (b) Tuesday
 (c) Wednesday
 (d) Sunday
- 1-j. Two pipes can fill a cistern separately in 10 hours and 15 hours. They can together fill the cistern in (CO5,K3) 1
 (a) 9 hours
 (b) 6 hours
 (c) 7 hours
 (d) None of these

2. Attempt all parts:-

- 2.a. Write the Cauchy's Riemann Equation in polar form. (CO1,K1) 2
- 2.b. Evaluate: $\oint_C \frac{z^2 - z + 1}{z - 2} dz$; where $C \equiv |z - 1| = \frac{1}{2}$. (CO2, K3) 2
- 2.c. Solve the PDE: $(D - 5D' + 1)^2 z = 0$ (CO3,K3) 2
- 2.d. Find inverse z - Transform of $F(z) = \frac{z}{z + 1}$. (CO4,K3) 2
- 2.e. At what time between 2 and 3 o'clock will the hands of a clock be together? (CO5,K2) 2

SECTION-B

30

3. Answer any five of the following:-

- 3-a. Discuss the analyticity of the function $f(z) = z\bar{z}$? (CO1,K3) 6
- 3-b. Find the bilinear transformation which maps the points $z = 0, -1, i$ into the points $w = i, 0, \infty$ respectively. (CO1,K3) 6
- 3-c. Evaluate $\oint_C \frac{e^z}{(z-1)(z-2)} dz$; $C \equiv |z| = 3$. (CO2,K3) 6
- 3-d. Evaluate $\oint_C \frac{4z^2 - 4z + 1}{(z-2)(z^2 + 4)} dz$; $C \equiv |z| = 1$. (CO2,K3) 6
- 3.e. Solve the PDE: $4r - 4s + t = 16 \log(x + 2y)$. (CO3,K3) 6
- 3.f. Find the Z - Transform of $\sin ak$, $k \geq 0$. (CO4,K4) 6
- 3.g. To complete a piece of work, Samir takes 6 days and Tanvir takes 8 days alone respectively. Samir and Tanvir took Rs. 2400 to do this work. When Amir joined them, the work was done in 3 days. Find the amount was paid to Amir? (CO5,K3) 6

SECTION-C

50

4. Answer any one of the following:-

4-a. Examine the nature of the function

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$$f(z) = \left\{ \frac{x^3 y(y - ix)}{x^6 + y^2}, z \neq 0 \right\}, f(0) = 0,$$
 , prove that $\frac{f(z) - f(0)}{z} \rightarrow 0$ as $z \rightarrow 0$ along any radiusvector but not as $z \rightarrow 0$ in any manner and also that $f(z)$ is not analytic at $z = 0$.

(CO1,K3)

4-b. Show that the function $v(x, y) = e^x \sin y$ is harmonic. Find its conjugate harmonic function $u(x, y)$ and the corresponding analytic function . (CO1,K3)

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5. Answer any one of the following:-5-a. Expand $\frac{1}{(z+2)(z+3)}$ for the following

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(i) $|z| < 2$ (ii) $|z| > 3$ (iii) $2 < |z| < 3$

(CO2,K3)

5-b. Evaluate $\int_0^{2\pi} \frac{d\theta}{5 - 4\cos\theta}$, Using contour integration . (CO2,K3)

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6. Answer any one of the following:-6-a. Solve $(D^2 - 2DD' - 15D'^2)z = 12(x+y)$. (CO3,K3)

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6-b. Solve the PDE : $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} - 2u$ subject to the condition : $u(x, 0) = 10e^{-x} - 6e^{-4x}$

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by method of separation of variables.

(CO3,K3)

7. Answer any one of the following:-7-a. Solve by z - transform: $y_{k+2} + 4y_{k+1} + 3y_k = 3^k$; $y_0 = 0, y_1 = 1$. (CO4,K3)

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7-b. Find Fourier cosine transform of e^{-x^2} . (CO4,K3)

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8. Answer any one of the following:-

8-a. (i) A and B can together finish a work in 30 days. They worked together for 20 days and then B left. After another 20 days, A finished the remaining work. In how many days A alone can finish the job ?

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(ii) A man and a boy together can do a certain amount of digging in 40 days. Their speeds in digging are in the ratio of 8 : 5. How many days will the boy take to complete the work if engaged alone? (CO5,K3)

8-b. (i) A, B, C, D and E are sitting on a bench. A is sitting next to B, C is sitting next to D, D is not sitting with E who is on the left end of the bench. C is on the second position from the right. A is to the right of B and E. A and C are sitting together. In which position A is sitting?

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(ii) 6 Boys are sitting in a circle and facing towards the Centre of the circle. Rajeev is sitting to the right of Mohan but he is not just at the left of Vijay. Suresh is between Babu and Vijay. Ajay is sitting to the left of Vijay. Who is sitting to the left of Mohan? (CO5,K2)

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