Printed Page:-04 **Subject Code:- AAS0104**

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: I - THEORY EXAMINATION (2024 - 2025)

Subject: Mathematical Foundations-I

Time: 3 Hours **Max. Marks: 100**

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. *This Question paper comprises of* ***three Sections -A, B, & C.*** *It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.*

2. *Maximum marks for each question are indicated on right -hand side of each question.*

3. *Illustrate your answers with neat sketches wherever necessary.*

4. *Assume suitable data if necessary.*

5. *Preferably, write the answers in sequential order.*

6. *No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.*

SECTION-A 20

1. Attempt all parts:-

1-a. If the eigen values of a matrix A are 4, 5, 7 then write the eigen values of A^{-1} are. (CO1,K1) 1

(a) $4, 5^2, 7^3$

(b) 4,5,7

(c) 1/4, 1/5, 1/7

(d) none of these

1-b. The rank of the matrix $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ is (CO1,k1) 1

(a) 1

(b) 2

(c) 3

(d) none of these

1-c. In the vector space V if $a+b=b+a$; where $a, b \in V$ is called (CO2,k1) 1

(a) Associativity

(b) Additive inverse

(c) Commutative

(d) None of these

1-d. A subset W is called subspace of vector space V(F) for $a, b \in F$ and $\alpha, \beta \in V$, if 1

.

satisfy (CO2, K1)

(a) $a\alpha - b\beta \in V$

(b) $a\alpha \; x \; b\beta \in V$

(c) $a\alpha \div b\beta \in V$

(d) $a\alpha + b\beta \in V$

1-e. If $y = e^{m\cos^{-1}x}$ then $y_1(0)$ is (CO3,K2) 1

(a) $-me^{m\frac{\pi}{2}}$

(b) $me^{-m\frac{\pi}{2}}$

(c) $me^{m\frac{\pi}{2}}$

(d) None of these

1-f. The asymptotes parallel to the y-axis of the curve $y^2(2a - x) = x^3$ is (CO3,K1) 1

(a) 2a

(b) $x = a$

(c) $x = 3a$

(d) None of these

1-g. 1

If u and v are the function of x and y then value of $\frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)}$ is (CO4,K1)

(a) 1

(b) 0

(c) x.y

(d) u.v

1-h. 1

Percentage error in calculating the area of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ when error of 1% is made in measuring the major and minor axes is given by (CO 4, K2)

(a) 2%

(b) 1%

(c) 4%

(d) 10%

1-i. The average weight of nine mangoes increases by 20 g if one of them weighing 120 g is replaced by another. The weight of the new mango is (CO5,K1) 1

(a) 180 g

(b) 200 g

(c) 260 g

(d) 300 g

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1-j. Find the missing terms: 6, 5, 7, 12.5, 27, ? (CO5,K1) 1

(a) 69
(b) 57.5
(c) 67.5
(d) 69.5

2. Attempt all parts:-

2.a. Find the value of P for which the matrix $A=\begin{bmatrix} 3 & P & P \\ P & 3 & P \\ P & P & 3 \end{bmatrix}$ is of rank 1. (CO1,K1) 2

2.b. Show that the three vectors (1, 1, -1), (2, -3, 5) and (-2, 1, 4) of R^3 are linearly independent. (CO2,K2) 2

2.c. If $u=\log\left(\frac{x^4+y^4}{x+y}\right)$, *Show that* $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3$. (CO3, K2) 2

2.d. If $u=\frac{x+y}{x-y}, v=\frac{xy}{(x-y)^2}$, then find $\frac{\partial(u,v)}{\partial(x,y)}$. Are u and v functionally related ? (CO 4,K2) 2

2.e. In a certain code GUEST is written as 53@$2 and MEAN is written as 6@4#. How is SAME written in that code? (CO5,K1) 2

SECTION-B 30

3. Answer any five of the following:-

3-a. Show that the matrix $A=\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is a unitary matrix. (CO1,K3) 6

3-b. Find the inverse of the matrix by using elementary transformations, where $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. (CO1,K2) 6

3-c. Find the dimension and basis of the subspace spanned by (2, 4, 2), (1, -1, 0), (1, 2, 1) and (0, 2, 1) in $V_3(R)$. (CO2,K2) 6

3-d. If α and β are vectors in an inner product space then show that $\|\alpha+\beta\|^2+\|\alpha-\beta\|^2=2\|\alpha\|^2+2\|\beta\|^2$. (CO2,K2) 6

3.e. If $u=e^{xyz}$, then show that $\frac{\partial^3 u}{\partial x\,\partial y\,\partial z}=(1+3xyz+x^2y^2z^2)\,e^{xyz}$. (CO3,K3) 6

3.f. Expand $e^x\cos y$ about the point (0,0) up to the three degree terms. (CO4,K2) 6

3.g. If in a certain code "RANGE" is coded as 12345 and "RANDOM" is coded as 123678. Then the code for the word "MANGO" would be ? (CO5,K2) 6

SECTION-C 50

4. Answer any one of the following:-

4-a. Show that the system of equations $3x+4y+5z=a, 4x+5y+6z=b, 5x+6y+7z=c$, does not have solution unless $a+c=2b$. (CO1,K3) 10

4-b. Find the rank of the matrix $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ by reducing it to normal form. (CO1,K2) 10

5. Answer any one of the following:-

5-a. For a 3-dimensional space R^3 over the field of real numbers R, determine if the set $\{(2, -1, 0), (3, 5, 1), (1, 1, 2)\}$ is a basis. (CO2,K2) 10

5-b. Define inner product space. Then show that $u = (u_1, u_2), v = (v_1, v_2)$ *in* R^2 *defined by* $\langle u, v \rangle = 4u_1v_1 + 5u_2v_2$ is inner product space. (CO2,K3) 10

6. Answer any one of the following:-

6-a. If $u = f(r)$, where $r = \sqrt{x^2+y^2}$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$. (CO3,K2) 10

6-b. If $y = a\cos(\log x) + b\sin(\log x)$, then show that $x^2y_2 + xy_1 + y = 0$ and $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$ (CO3,K3) 10

7. Answer any one of the following:-

7-a. **u, v ,w are the roots of the cubic** $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ **in** λ **then find** $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. (CO4,K2) 10

7-b. Find the extreme values of the function $x^3 + y^3 - 3xy$. (CO4,K2) 10

8. Answer any one of the following:-

8-a. (i) In certain code language SERIES is coded as 5625 and PIPE is coded as 2116. How will WAP be coded in the same code language? 10

(ii) The average marks obtained by 22 candidates in an examination are 45. The average marks of the first 10 candidates are 55 and those of the last eleven are 40.The number of marks obtained by the eleventh candidate is ?

(iii) A candidate scores 25 % marks and fails by 30 marks, while another candidate who scores 50 % marks get 20 marks more than the minimum marks required to pass the examinations. Find the maximum marks for the examination. (CO5,k3)

8-b. (i) Pankaj purchased an item for Rs. 7500 and sold it at the gain of 24%. From that amount he purchased another item and sold it at the loss of 20%. What is his overall gain/loss? 10

(ii) The average of runs of a cricket player of 20 innings was 32. How many runs must he make in his next innings so as to increase his average of runs by 4 ? (

(iii) In certain code language, ROCK=47 and LATE=38. Find the code for FOOL. (CO5,K3)