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NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: I - THEORY EXAMINATION - (2024 - 2025)

Subject: Engineering mathematics I

Time: 3 Hours

Max. Marks: 100

General Instructions:

IMP: Verify that you have received the question paper with the correct course, code, branch etc.

1. This Question paper comprises of **three Sections -A, B, & C**. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.

2. Maximum marks for each question are indicated on right -hand side of each question.

3. Illustrate your answers with neat sketches wherever necessary.

4. Assume suitable data if necessary.

5. Preferably, write the answers in sequential order.

6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION-A

20

1. Attempt all parts:-

1-a. 1

If a matrix of order $m \times n$ can be expressed as $A = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$, the rank of A is (CO1,K1)

- (a) r
- (b) r+1
- (c) r-1
- (d) none of these

1-b. 1

If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$, then the eigen values of A^2 are (CO 1,K1)

- (a) 1, 7, 9
- (b) 1, 4, 9
- (c) 1, 1, 2
- (d) 3, 6, 9

1-c. 1

If $u = e^x(x \cos y - y \sin y)$, then find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ is (CO2,K2)

- (a) 2u
- (b) 4
- (c) 0

(d) $u = e^x(x \cos y - y \sin y)$

1-d. If highest power of x are constant then the Asymptote... (CO2,K1) 1

- (a) Parallel to x-axis
- (b) Parallel to y-axis
- (c) No. asymptotes parallel to x-axis.
- (d) No. asymptotes parallel to y-axis.

1-e. The point at which the function $x^3 - 4xy + 2y^2$, maximum or minimum is (CO3,K2) 1

- (a) $\left(\frac{4}{3}, \frac{2}{3}\right)$
- (b) $\left(\frac{4}{3}, \frac{1}{3}\right)$
- (c) $\left(1, \frac{1}{3}\right)$
- (d) $\left(\frac{4}{3}, \frac{4}{3}\right)$

1-f. Percentage error in calculating the area of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ when error of 1% is made in measuring the major and minor axes is given by (CO3,K1) 1

- (a) 2%
- (b) 1%
- (c) 4%
- (d) 10%

1-g. The value of $\Gamma(-1/2)$ is (CO4,K2) 1

- (a) $2\sqrt{\pi}$
- (b) $-2\sqrt{\pi}$
- (c) $-\sqrt{\pi}$
- (d) None of these

1-h. The value of $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} dx dy dz$ is (CO 4,K1) 1

- (a) $(e-1)^2$
- (b) $(e+1)^2$
- (c) $(e-1)^3$
- (d) None of these

1-i. In the college election, a candidate secure 62% of the votes and elected by a majority of 144 votes. The total number of votes polled is (CO5,K1) 1

- (a) 800
- (b) 925

- (c) 120
(d) 600

1-j. Find the missing term : 563, 647, 479, 815, ? (CO5,K2)

1

- (a) 672
(b) 386
(c) 279
(d) 143

2. Attempt all parts:-

2.a. For the given matrix $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, prove that $A^3 = 19A + 30I$. (CO1,K1) 2

2.b. Find the nth derivative of $y = \frac{1}{(2x+3)(3x-1)}$ (CO2,K2) 2

2.c. In $RI = E$, possible error in E and I are 20 % and 10% respectively then find the error in R. (CO3,K1) 2

2.d. Find the value of $\int_0^5 \int_{2-x}^{2+x} dy dx$. (CO 4,K1) 2

2.e. If out of 10 selected students for an examination, 3 were of 20 years age, 4 of 21 years and 3 of 22 years, then the average age of the group ? (CO5,K2) 2

SECTION-B

30

3. Answer any five of the following:-

3-a. Find the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ by elementary transformation. (CO1,K2) 6

3-b. Test the consistency of system of equation $10y + 3z = 0$, $3x + 3y + z = 0$, $2x - 3y - z = 5$, $x + 2y = 4$. (CO1,K1) 6

3-c. If $u = e^{xyz}$, then show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}$. (CO2,K2) 6

3-d. If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$, prove that (CO2,K2) 6

(i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$
(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \cos 3u \sin u$.

3.e. If $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_1 x_3}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$ then find the value of $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)}$. (CO3,K1) 6

3.f. Evaluate $\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}} dx$. (CO 4,K2) 6

- 3.g. In certain code language 'si po re' means 'book is thick', 'ti na re' means 'bag is heavy', 'ka si' means 'interesting book' and 'de ti' means 'that bag'. What should stand for 'that is interesting' in that code language? (CO5,K1) 6

SECTION-C

50

4. Answer any one of the following:-

- 4-a. Find the eigen values and eigen vectors of a matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. (CO1,K2) 10

- 4-b. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence compute A^{-1} . Also evaluate $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$. (CO1,K3) 10

5. Answer any one of the following:-

- 5-a. If $y = e^{a \sin^{-1} x}$, then prove that $(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} - (n^2 + a^2) y_n = 0$ and find $y_n(0)$. (CO2,k3) 10

- 5-b. If $u = f(r)$, where $r = \sqrt{x^2 + y^2}$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$. (CO2.K2) 10

6. Answer any one of the following:-

- 6-a. If u, v, w are the roots of the cubic equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (CO3,K3) 10

- 6-b. Use the method of Lagrange's multiplier to find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. (CO3,K2) 10

7. Answer any one of the following:-

- 7-a. Evaluate by changing the order of integration $\int_0^a \int_{x^2/a}^{2a-x} xy \, dy \, dx$. (CO 4,K3) 10

- 7-b. Apply Dirichlet's integral to find the volume and mass contained in the first octant solid region of the curve $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, if the density at any point is $\rho(x, y, z) = kxyz$. (CO4) 10

8. Answer any one of the following:-

- 8-a. (i) A shopkeeper allows a 10% discount of to his customers and still gains 20%. Find the marked price of the article which costs Rs 450. 10
(ii) The average of marks of 17 students in an examination was calculated as 71. But it was later found that the mark of one student had been wrongly entered as 65 instead of 56 and another as 24 instead of 50. Find the correct average?
(iii) If the numerator of a fraction is increased by 20% and its denominator is

decreased by 10%, the fraction becomes $\frac{3}{2}$. Find the original fraction. (CO5,K1)

8-b. (i) If the price of an item is decreased by 10% and then increased by 10%, then what is the net effect on the price of the item? 10

(ii) The average marks obtained by 40 students of a class is 86. If the 5 highest marks are removed and the average reduced by one mark. Find the average marks of the top 5 students?

(iii) Find the missing terms: 1, 2, 6, 7, 21, 22, 66, 67, ?
(CO5,K1)

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